Preliminary Analysis of 3-D Scramjet Flowpath with MGD Control

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Magnetogasdynamic (MGD) interactions are simulated to control two separate aspects of scramjet flowfields. First, the possibility of mitigating surface loads in a Type IV shock-on-cowl lip flow is explored with a radially decaying magnetic field. In this electrodeless configuration, two different conductivity variations are imposed, respectively a) to retard the supersonic impinging jet directly near the body surface, and b) to leverage small displacements of the principal triple point. Results suggest that heat transfer reduction can be accomplished with the Lorentz force mechanism, albeit at relatively high values of the interaction parameter. The second approach is more effective than the first—a lower interaction parameter is needed to obtain similar load mitigation and a Type III pattern is obtained—but requires imposed perturbations at larger distances from the surface.

The second part of the paper describes a 3-D computation of the flow path in a simulated scramjet with a segmented electrode configuration. Elements of the device include pitch-plane compression followed successively by an MGD-assisted horizontal diffuser, an isolator/combustor segment and finally an MGD-assisted nozzle. Particular emphasis is placed on problem setup, developing analysis techniques and on examining trends. The established flowfield is described first without and then with MGD control. For the latter case, electromagnetic aspects including current, electric and Lorentz force fields are elucidated. Flow separation yields significant three-dimensional effects, and together with viscous near-wall phenomena, modify the MGD interaction substantially relative to the 2-D or inviscid cases. The effectiveness of the generator in reducing Mach number at the entrance to the isolator/combustor is demonstrated at an interaction parameter of unity. In both generator and accelerator, evaluation of energetic interactions indicate promising trends in terms of the ratio of reversible to irreversible contributions.

1 Introduction

The potential of electromagnetic interactions to alleviate or eliminate many of the pressing obstacles in accomplishing sustained hypersonic flight has led to considerable renewed attention on these techniques which can reduce local gradients, induce or suppress fluid dynamic bifurcations, or can provide beneficial force and energy interactions between the fluid and the vehicle. In recent years, a wide range of theoretical, experimental and numerical efforts1–8 have refined current understanding of parameters that are necessary to accomplish the desired objectives of reducing thermomechanical loads and improving propulsion (typically scramjet) efficiencies. Key parameters that have been identified include among others, choice of ionization technique (the flow will typically not be thermally ionized), energy budgets, magnetic field magnitudes and physical issues such as electrode design and placement.

Several promising concepts have been proposed and explored1,2,7 particularly in reference to scramjet performance. These address control of shock location for inlet mass-capture, mixing enhancement, and on a larger scale, the MHD-energy bypass concept, where energy extracted efficiently from an inlet is employed for other on-board activities or to enhance thrust. This paper considers MGD control of two different problems, the first to alleviate high loads generated on the cowl-lip at design conditions, and the second to modify the three-dimensional scramjet flowpath.

The principal mechanisms of magnetogasdynamics are Lorentz forces and energy interactions, which include reversible as well as irreversible components. The impact on the flow may be characterized by several non-dimensional parameters, including the magnetic Reynolds ($R_m$) and pressure numbers ($R_p$) and their product, the interaction parameter ($Q$). The aerospace environment of interest is characterized by relatively low conductivity ($R_m << 1$), which must be compensated for by a large magnetic field ($R_m >> 1$) to obtain reasonable fluid-magnetic field interaction ($Q \sim O(1)$). These considerations bring to fore many engineering issues, such as those listed above, and also guide the development of the theoretical formulation. At low magnetic Reynolds numbers, the effect

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American Institute of Aeronautics and Astronautics Paper 2002-2134
Problem II: MHD Energy-Bypass

Fig. 1 Overview of problems examined

of the induced part of the magnetic field is negligible. This permits the use of the source term formulation described in § 2, in which the electromagnetic interaction is incorporated as a source term to the (3-D) Navier-Stokes equations. In addition to the fluid velocity, this term contains the magnetic field which is imposed, the electrical conductivity, which is specified (heuristic), the current vector, which is described phenomenologically with the generalized Ohm's law and the electric field vector, which is obtained from Kirchhoff's first law. Although turbulence, Hall-effect and ion-slip terms are incorporated in the formulation, they are not employed in the present calculations.

The numerical method employed has been developed in several prior publications and is outlined in § 3. The solution procedure adapts techniques developed for fluid dynamics (see e.g., Ref. 9). Briefly, a finite-difference implementation in general curvilinear coordinates is used in which the viscous terms of the fluid equations are evaluated with a combination of high-resolution upwind schemes while the viscous terms are centered. The current continuity equation for electric potential is discretized with high-order compact-difference and filtering components. The fluid and Poisson equations are marched in time and pseudo-time respectively with implicit approximately factored methods. Special attention is paid to developing boundary conditions not only for the fluid dynamic variables but also for electrode and insulator surfaces.

The two problems investigated may be put into context by considering the simulated scramjet device shown in Fig. 1. This dual-plane compression system, adapted from the concept proposed in Ref. 10, incorporates many of the phenomena of concern to scramjet designers with minimal geometrical complexity. The first problem arises from the need, based on mass cap-

ture considerations, to focus the external compression shock system on the cowl lip, which is typically represented by a curved surface of relatively small radius of curvature. The resulting interaction of the ramp (impinging) shock with the bow shock yields one of several complex patterns which have been classified by Edney into six types. Of these, the so-called Type IV pattern, characterized by the impingement of a supersonic jet on the surface of the body, typically yields the highest surface pressures and heat transfer rates. Section 4.1 explores magnetogasdynamic control of this interaction under two specified electrical conductivity distributions combined with a radially diminishing magnetic field.

The second aspect of the configuration of Fig.1 concerns elements of the so-called MHD bypass concept, which has been described in Refs. 12, 13 (for example) and discussed in system level detail in Refs. 2, 14. The proposal first calls for the conversion of kinetic energy of the inlet flow into electromagnetic energy, in a manner that not only reduces losses compared to the case where such compression is accomplished through shock waves, but also provides energy that can then be utilized for additional thrust through MGD acceleration in the nozzle downstream (or for other onboard functions). The entire 3-D flowpath concept, including uncoupled generator and accelerator components, is simulated in Section 4.2, with emphasis on problem setup, solution and trend detection, by considering a full three-dimensional segmented-electrode configuration operating in Faraday mode.

2 Governing Equations

The low magnetic Reynolds number (or source term) form is obtained by coupling the Navier-Stokes equations with Lorentz force and energy interaction terms:

\[
\frac{\partial \rho^*}{\partial t^*} + \nabla^* \cdot (\rho^* \mathbf{U}^*) = 0 \tag{1}
\]

\[
\frac{\partial \rho^* \mathbf{U}^*}{\partial t^*} + \nabla^* \cdot [\rho^* \mathbf{U}^* \times \mathbf{U}^* + \rho^* \mathbf{B}^*] - \frac{1}{Re} \nabla^* \cdot \tilde{\mathbf{r}}^* = \mathbf{Q} \left( \mathbf{j}^* \times \mathbf{B}^* \right) \tag{2}
\]

\[
\frac{\partial \rho^* \mathbf{e}^*}{\partial t^*} + \nabla^* \cdot \left[ (\rho^* \mathbf{e}^* + \rho^* \mathbf{U}^* \cdot \mathbf{U}^*) - \frac{1}{Re} (\mathbf{U}^* \cdot \mathbf{r}^*) - \frac{1}{(\gamma - 1) Pr M^2 Re Q_{ht}} \right] = \mathbf{Q} \left( \mathbf{E}^* \cdot \mathbf{j}^* \right) \tag{3}
\]

The superscript * denotes a non-dimensional quantity, \( t \) is the time, \( \rho \) is the density, \( \mathbf{U} = \{u, v, w\} \) is the velocity vector, \( p \) is the pressure, \( \mathbf{B} \) and \( \mathbf{E} \) are the magnetic and electric fields respectively, \( Q_{ht} \) is the heat conduction term, \( \mathbf{r} \) is the shear stress tensor and \( e = \frac{\mathbf{p} + \mathbf{e}^2}{\gamma - 1} \) is the total energy. The transport properties are the molecular viscosity \( \mu \), obtained with Sutherland's law, and the electrical conductivity \( \sigma \) which is

American Institute of Aeronautics and Astronautics Paper 2002-2134
presently specified. In addition to the Reynolds number, $Re = \frac{\mu_{\text{ref}} U_{\text{ref}} L_{\text{ref}}}{\mu_{\text{ref}}}$, Mach number, $M$, and Prandtl number, $Pr = \frac{\mu_{\text{ref}} C_p}{k_{\text{ref}}}$, an interaction parameter $Q = \sigma_{\text{ref}} B_{\text{ref}}^2 L_{\text{ref}} f(\frac{\mu_{\text{ref}} U_{\text{ref}}}{\mu_{\text{ref}}})$ appears in the source term. In the subsequent discussion, the superscript (*) will be dropped and all quantities will be assumed to be non-dimensional unless explicitly otherwise stated.

The current vector, $\mathbf{j}$, is obtained from the phenomenological form of the generalized Ohm’s law:

$$\mathbf{j} = \tilde{\sigma} \cdot \left[ \mathbf{E} + \mathbf{U} \times \mathbf{B} \right]$$  \hspace{1cm} (4)

In the general case, the tensor conductivity $\tilde{\sigma}$ includes the Hall and ion-slip parameters. The reasonable assumption has been made that the term due to electron pressure gradient is negligible (see e.g., Ref. 16). Detailed expressions for the source terms as well as the conductivity have been provided in Ref. 17 and are not repeated here.

The electric field $\mathbf{E}$ is determined from the current continuity condition:

$$\nabla \cdot \mathbf{j} = 0 \hspace{1cm} (5)$$

Introducing a scalar potential, $\mathbf{E} = -\nabla \phi$, the equation solved is:

$$\nabla \cdot [\tilde{\sigma} \cdot (\nabla \phi)] = \nabla \cdot \left[ \tilde{\sigma} \cdot \mathbf{U} \times \mathbf{B} \right] \hspace{1cm} (6)$$

3 Numerical details

Algebraic manipulation permits the fluid dynamic equations to be written in flux vector form:

$$\frac{\partial \mathbf{X}}{\partial t} + \frac{\partial \mathbf{F}_l}{\partial \xi} + \frac{\partial \mathbf{G}_l}{\partial \eta} + \frac{\partial \mathbf{H}_l}{\partial \zeta} = \frac{\partial \mathbf{F}_V}{\partial \xi} + \frac{\partial \mathbf{G}_V}{\partial \eta} + \frac{\partial \mathbf{H}_V}{\partial \zeta} + \mathbf{S} \hspace{1cm} (7)$$

where a general curvilinear coordinate transformation has been introduced, $x = x(\xi, \eta, \zeta)$, $y = y(\xi, \eta, \zeta)$ and $z = z(\xi, \eta, \zeta)$, in order to facilitate the treatment of complex configurations. The solution vector, $\mathbf{X}$ is $1/J\{\rho, \rho u, \rho v, \rho w, \rho e\}$, where $J$ is the Jacobian of the coordinate transformation. $F_l$, $G_l$, and $H_l$ contain terms relevant to inviscid fluxes, $F_V$, $G_V$, and $H_V$ include effects due to viscosity, and $S$ is the source term containing electromagnetic interaction terms. The various vectors of Eqn. 7 have been detailed in Refs. 18, 19.

Since the influence of the magnetic field is restricted to the source term in the present low $R_a$ approach, conventional CFD techniques including both the Roe and van Leer schemes have been incorporated together with the MUSCL scheme to obtain up to nominal third order accuracy. Solution monotonicity is assured with the harmonic limiter described in Ref. 22.

The equation for the electric potential, Eqn. 6, may also be written in the scalar equivalent of the form of Eqn. 7 by introducing a pseudo-time term. Spatial discretization is accomplished with fourth- or sixth-order compact differences as described in Ref. 6. Since the flow field contains shocks, high-order differencing the source term in Eqn. 6 can introduce numerical instabilities. In such cases, smoothing is applied with a Padé-type filter.

Both the Navier-Stokes and electric potential equations are integrated in time with the approximately factored Beam-Warming-type method as described in Ref. 17.

Boundary conditions for the fluid equations are imposed in a straightforward fashion. On all solid surfaces, the no-slip condition is enforced, the pressure gradient is assumed to be zero and the case-dependent wall temperature is specified as below. For all cases considered, the downstream boundaries are predominantly supersonic. Consequently, the zero gradient condition is applied. At inflow boundaries, the flow vector is specified. Detailed conditions for the electric potential at insulators and electrodes have been described in Ref. 6. The sharp variation of $\phi$ on boundaries at electrode/insulator junctions gives rise to numerical instabilities which propagate into the domain. To suppress these, the boundary values of $\phi$ are also filtered with the low-pass procedure described in Ref. 6. Additional specific issues are discussed later in the context of each problem.

4 Results

The simulated scramjet configuration shown in Fig. 1 consists sequentially of a forebody pitch-plane compression, sidewall compression, a constant area duct as isolator/combustor and a nozzle. Considerations factored into developing this configuration will be presented in § 4. The two aspects address in turn the cowl heating problem, which affects a small region around the lip, and the global issue of MHD energy-bypass operation. Although the two problems are parts of the same configuration, they are treated separately and flow parameters chosen are not identical.

4.1 Control of Type IV interaction

The catastrophic impact of high thermal loads is a major concern in high-speed flight. Regions of large heat transfer rates occur not only at the nose of the vehicle but also downstream where complex shock-shock and shock-boundary layer interactions occur. One such situation occurs on inlet cowl at the optimum mass-capture design condition, in which the vehicle forebody shocks converge on the inlet lip and interact with the lip shock. The pattern obtained has been classified by Edney11 into six different types; the Type IV pattern to be described shortly yields the largest localized pressure and heat transfer rates on the cowl lip.

American Institute of Aeronautics and Astronautics Paper 2002-2134
The flow parameters chosen are patterned after the experimental results of Ref. 23. The cowl lip is modeled as a cylinder of radius, $R = 0.038\,m$, the Mach number is $8.03$, $T_{\infty} = 111.6\,K$ and $Re = 39,222/m$. The impinging shock corresponds to a flow deflection angle of $12.5^\circ$. Thus $M_2 = 5.25$, $p_2/p_{\infty} = 7.13$ and $\rho_2/\rho_{\infty} = 3.33$ where subscript 2 refers to conditions downstream of the shock. The shock locus is described by the equation, $y = 0.327x + 0.415$. A posteriori analysis indicates these conditions do not yield significant high-temperature effects.

The grid employed, and aspects of baseline flow field (no MGD) are shown in Fig. 2, where note that the impinging shock orientation has been reflected with respect to the horizontal as compared to the schematic of Fig. 1. The mesh consists of $101 \times 151$ points, which are clustered near the expected supersonic jet in the circumferential ($\theta$) direction and near the body-surface in the $r$ direction. Boundary conditions are applied in the manner described above, with the wall temperature specified at $294.4\,K$. The impinging shock is imposed by using the jump condition to specify the flow at and beneath the intersection of the shock with the outer boundary. The Roe and van Leer schemes are employed in the body-normal and circumferential directions respectively. This approach has been shown to provide accurate results without the complications associated with the formation of the so-called “carbun-cle”.

Contours of Mach number, shown in Fig. 2(b), highlight the basic features of the interaction. The Type IV pattern is obtained when the impinging shock intersects the nearly normal part of the cowl shock. Two triple points ($TP$), marked $TP1$ and $TP2$, are formed from which emanate two shear layers. The nearly body-normal supersonic jet arising between these two $TP$s passes through a terminating jet shock and strikes the body to yield sharp local pressure and heat transfer peaks. The stagnated fluid then accelerates around the body to supersonic speeds, encountering a shock/expansion train and forming a subsonic pocket between the bow shock and the boundary layer. All of these well known features are captured accurately by the current technique. The known unsteadiness of the interaction,\textsuperscript{24,25} manifested in part by an oscillating terminating jet shock is also observed. Figure 2(c) shows heat transfer loads at several time instants $5T$ apart (here $T$ is the characteristic time, $T = R/U_{\infty}$). The abscissa is the angle measured in degrees from the horizontal (see Fig. 2(b)). Note the sharp peak in the region centered about $\theta' \sim -20^\circ$ where the jet impinges on the cylinder. Although the peak value oscillates only modestly, larger oscillations are observed on either side of the maximum where complicated near-wall phenomena dominate.

In an effort to control the flow, a magnetic field consistent with current directed normal to the plane of the flow, i.e., along the axis of the cylinder is imposed. The field, shown in Fig. 3(a), is thus circumferential and decays in the radial direction:

$$\vec{B}(r, \theta') = \frac{B_0}{r} (-\sin\theta' i + \cos\theta' j)$$  \hspace{1cm} (8)

where $\theta'$ is measured relative to the $+x$-axis (see Fig. 2(b)). The normalized value is unity on the surface while the physical value is established in terms of the interaction parameter as discussed below.

Two different distributions are considered for the conductivity ($\sigma$) as shown in Fig. 3(b) and (c) respectively. These are generated with modified Gaussians

American Institute of Aeronautics and Astronautics Paper 2002-2134
(MG) of the form

$$\sigma_1(A, a, s, s_0, n) = Ae^{-\alpha(s-s_0)^n} \quad (9)$$

where $A$ is the peak (set to unity for normalized values), $a$ determines the width as below, $s$ is the distance variable, centered at $s_0$ and $n$ is the exponent, which is set to 6. The width, $a$, is determined by specifying a value of conductivity, $\epsilon$ at a specified distance $\delta$ from the center $s_0$:

$$a = \frac{1}{\delta^n} \log\left(\frac{A}{\epsilon}\right) \quad (10)$$

This general form is consistent with that produced by a non-equilibrium ionization method proposed for aerospace applications (see Ref. 14). The impact of flow gradients on this distribution has been ignored in the present effort.

The first $\sigma$ distribution, denoted $\sigma - A$, encompasses the jet as shown in Fig. 3(b) and is obtained as the product of two MGs. In the first MG, $s$ is the radial distance, $s_0 = 1.2$, $\epsilon = 0.1$, $\delta = 0.2$ (normalized distance units), which yields a surface $A$ of 0.1. The second MG determines the circumferential extent of the distribution, with $s$ associated with $\theta'$, $s_0 = 3.5$ (radians), $\epsilon = 0.1$ and $\delta = 1.0$ (radians).

The second $\sigma$ variation, $\sigma - B$, shown in Fig. 3(c), is designed to modify the triple point, $T P 1$, whose location is correlated to the type of interaction produced and consequent surface loads, as well as to exploit the relatively low local $\rho U$ value in the far field. This circumferential region lies in the undisturbed freestream, near $T P 1$. The radial $MG$ has the parameters $\epsilon = 0.1$, $\delta = 0.41$. The distance parameter $s$ is linked to the radial distance from $s_0$, which is located at $x = -2.26$, $y = -0.054$. The implicit $\theta$ directional MG is unity.

Magnetic control is explored at various values of the nominal interaction parameter, $Q = \sigma_{ref} B_{ref} L_{ref}/\rho_{ref} U_{ref}$. In this, $\sigma_{ref}$ is the peak conductivity in the interaction region, $B_{ref}$ is the magnitude of the $B$ field at the surface of the cylinder and $\rho_{ref}$ and $U_{ref}$ are values in the far field. Local values for $B$ are lower than $B_{ref}$ because of radial decay, especially for $\sigma - B$, and appropriate values may be taken to be 0.8 and 0.45 respectively for $\sigma - A$ and $\sigma - B$. An effective interaction parameter, $Q_{eff}$, based on local values is therefore established to characterize interaction strength. Three values of $Q_{eff}$, 0.64, 6.4 and 12.8 are simulated for $\sigma - A$. For reasons outlined later, lower values are chosen for $\sigma - B$, 0.2, 1, 2 and 4. In physical terms, wind tunnel conditions dictate, $\rho_{ref} = 1.9 \times 10^{-3} \text{kg/m}^3$, $U_{ref} = 1700 \text{m/s}$ and $L_{ref}$ is the cylinder radius, 0.0381m, which yields $\sigma_{eff} B_{eff}^2 \sim 85 Q_{eff} T^2 - \text{mho/m}$. (Although $\rho_{ref} U_{ref}$ is representative of $\sigma - B$, local values for $\sigma - A$ are on average higher but are more difficult to characterize and this variation is thus ignored in obtaining $Q_{eff}$.) As an example, $Q_{eff} = 1$ requires

Figure 3: Problem setup for MGD flow control of Type IV interaction.

$$Q_{eff} B_{eff}^2 = 85 T^2 - \text{mho/m}, \text{ which may be obtained with } \sigma \sim 2 \text{mho/m and } B \sim 7T. \text{ Although these are large values, they are not inconsistent with those proposed in system level studies. Obviously much larger } Q_{eff} \text{ parameters such as some employed here must be considered presently out of reach, unless the value of } \rho_{ref} U_{ref} \text{ is lower. Since the Hall-effect introduces significant cross and secondary flows in this configuration, which violate the 2-D assumption, the Hall (and ion-slip) parameters were set to zero. The present results therefore constitute a baseline for future work in which these restrictions will be lifted.}

The effect of MGD forces on surface pressure is
shown in Fig. 4 for both σ variations. The ordinate has been normalized with peak values in the absence of control. For σ = A (Fig. 4(a)), the change in peak pressure is monotone: it drops sharply with Q_eff and its location moves towards the lower part of the cylinder. For the largest Q_eff, the peak pressure is reduced to only 20% of its no MGD value. However a modest pressure rise is evident on the upper part of the cylinder, where the boundary layer develops after impingement. Results with σ = B, Fig. 4(b), do not display monotonic change. Although the peak here too moves towards the lower part of the cylinder, its magnitude increases at lower values of Q_eff. Only for the highest Q_eff does the peak drop by about 20%. In both σ distributions, the undulations observed away from the peaks are reduced. Note that the overall reduction in integrated pressure with σ = A does not necessarily correspond to a drop in drag because the effect of the magnetic pressure, which depends on B^2, has not been factored: this value may increase depending on how σ_eff B^2_eff is specified (while ensuring that Rσ remains low to satisfy the chosen theoretical formulation) to obtain the specified Q_eff.

The MGD effect on heat transfer is shown in Fig. 5 where again values are normalized with the no-MGD peak. It is evident that the impact on heat transfer is less dramatic than on pressure. For σ = A, Fig. 5(a), Q_eff = 1 has minimal effect while values of 6.4 and 12.8 decrease peak values by roughly 10% and 20% respectively. However, in these cases, the heat transfer profile becomes flatter and the local rate actually increases in many regions of the surface. For σ = B, Fig. 5(b), although the peak is undiminished at Q_eff = 0.2, its location moves downward as before. Increasing Q_eff shows a monotonic decline in peak values and undulations in regions away from the peak also diminish. Although σ = B also flattens the profile, the effect is less pronounced than with σ = A. It is evident that the flow is far more sensitive to σ = B than to σ = A and that the former is more effective than the latter. For example, the reduction in heat transfer at Q_eff = 4 with σ = B is comparable to that obtained with σ = A at the much higher Q_eff of 12.8. Clearly, details of the configuration are crucial in determining control effectiveness as discussed further below.

Current magnitude contours are shown in Fig. 6 for the highest Q_eff of each σ variation together with the respective shock-envelopes. Consistent with a vanishing electric field, the z-directional current is dependent on local σ and $\vec{U} \times \vec{B}$ and points into the plane of the flow nearly everywhere. Naturally, non-zero currents are only observed in conducting regions of the flow, which as noted earlier are assumed to be decoupled from the established flow pattern. For σ = A, highest values are observed in the supersonic jet and immediately downstream of the lower part of the distorted bow shock while for σ = B, maxima occur immediately downstream of the new location of TP1.

Figure 7 exhibits the corresponding Lorentz force magnitude with contours and select force lines. The variation is generally similar to the current though local maxima are displaced because of the additional cross-product with the spatially varying magnetic field. The magnetic force is directed radially outwards in both conductivity variations, consistent with a retardation of the freestream flow. In σ = A however, the fluid recovering from the stagnation on the upper part of the cylinder exhibits a nearly vertical velocity vector. The current here is relatively small but is directed in the +z direction and a small component of force is observed towards the wall and may correlate with the modest increase in surface pressure observed in Fig. 4(a) in the region θ > 0 at higher Q_eff values.

Although not explored in detail, it is noted that the magnetic interaction showed a reduction in the level of the unsteadiness of the interaction. This observation may be attributed to the damping effect of the magnetic force. The impact of the two control con-
figurations on the flow field is summarized in Fig. 8, which exhibits Mach contours at $Q_{ref} = 6.4$ for $\sigma - A$ and $Q_{ref} = 2$ for $\sigma - B$. In all cases, the shock envelope increases in size and a limited amount of shock smearing is observed near TP1 for $\sigma - A$ and TP2 for $\sigma - B$ respectively. More crucially, it is evident that $\sigma - A$ does not affect the basic nature of the interaction – the features of the Type IV pattern are still clearly visible though the wall-normal jet has been diffused. However, $\sigma - B$ now yields a Type III interaction, characterized by a shear layer which grazes the surface. This pattern modification is closely linked to the change in location of TP1. Although many issues contribute to the choice of control configuration, not least the difficulty of generating adequately conducting regions of proper shape and location, these results suggest nonetheless that magnetic field effects can be leveraged most profitably by modifying, if possible, the fluid dynamic bifurcation ($\sigma - B$), rather than on local gradient modification ($\sigma - A$).

American Institute of Aeronautics and Astronautics Paper 2002-2134
Fig. 7 Lorentz forces in MGD control of Type IV interaction
will be termed the vertical diffuser. At this point, sidewalls are converged at 4° inclination each, to obtain spanwise compressions - this segment will be called the horizontal diffuser in the subsequent discussion. Sidewall compressions terminate where these crossing shocks reach the opposite walls. At this point, a constant area duct is introduced to serve as an isolator and combustor, which is followed by a nozzle with 15° thrust surfaces, in which fluid is permitted to expand freely. The freestream flow parameters are assumed to be $M = 8$, $T_\infty = 250K$, $Re = 1.6 \times 10^6$ (based on width of configuration at the inlet, which is set at 0.6m).

To explore MGD effects, four segmented-electrode pairs are placed in the generator and the accelerator respectively, as depicted in Fig. 10. The generator segment is placed in the horizontal diffuser part of the configuration while the accelerator is placed in the nozzle, downstream of the combustor. The potentials established on electrode surfaces depend upon load factors and are determined from the mean velocity and magnetic fields as described later.

The simplicity of the configuration facilitates the principal goal of investigating numerical and problem setup issues as well as to explore and confirm expected

American Institute of Aeronautics and Astronautics Paper 2002-2134
trends. It is anticipated that the features of this simulated scramjet configuration will evolve as further understanding is acquired. In addition to laminar, frozen flow and absence of Hall-effect, two additional simplifications invoked are that there is no heat input in the combustor and electrode sheath effects have been neglected.

Boundary conditions are applied in the manner described earlier but with one modification. To prevent extremely high wall temperatures, the value specified is the smaller of the adiabatic value or 1000$K$. Boundary conditions for the electric field are linked to the computed flow field and are described later.

To facilitate the difficult task of describing the 3-D flowfield, several sectional cuts will be employed in addition to perspective views. Fig. 11 depicts these and also serves to exhibit the grid, which is formed by stacking non-uniform $x = \text{constant}$ Cartesian planes in the streamwise direction. The total grid size is $201 \times 41 \times 41$ – the figure shows a coarser mesh obtained by plotting only every third line in the $x$ direction and every other line in the $y$ and $z$ directions. Although this is a relatively coarse mesh, points are clustered near the walls to resolve the boundary layer, while shock waves which dominate the flow away from the walls are typically captured in only one or two zones by the present high-resolution method. The planes employed to depict the flow consist of the $j$ (Fig. 11(a)) and $k$, 11(b), computational mid-planes. The latter is planar because of vertical symmetry while the former deviates from horizontal in the vertical diffuser section by a small angle. Several $i$-planes of the type shown in Fig. 11(c) are also employed.

4.2.2 Baseline flowfield

The flowfield without MGD control is described first. Figure 12 shows the pressure contours on the $j$-midplane. The central figure shows the entire domain while the upper and lower subfigures show details in the horizontal diffuser and downstream regions respectively. The cut shows sharp increases in pressure at the location where the ramp shock and its reflection intersect the plane. Although not evident in pressure contours, the flow in the region between the cowl lip and the diffuser becomes three-dimensional because of the development of sidewall boundary layers which interact with these shocks. The subsequent rise in pressure in the diffuser (top figure) shows the crossing shock pattern. However, because of the aforementioned sidewall interaction in the vertical diffuser, as well as the expansion originating at the end of the fuselage, the flow at the entrance to the converging segment is considerably more complex than that encountered in the standard double-fin configuration of Ref. 26. Note also the diffuse nature of the pressure contours near sidewalls, as a result of the shock/boundary layer interaction associated with shocks from the opposite sidewall. The pressure field in the isolator/combustor section shows the shock-train in which streamlines go through complicated alternating shocks and expansions. Gradients in the streamwise direction gradually diminish until the trailing nozzle where the pressure reduces rapidly as the flow is permitted to expand.

Despite the relatively mild angles, the interaction of shocks in the horizontal diffuser with boundary layers on the upper and lower walls gives rise to threedimensional boundary layer separation. Various as-
Inviscid values typically employed in simpler formulations. Note that although separation is clearly evident, there is no significant region of reversed flow. This observation highlights the fundamentally 3-D nature of the interaction.

Figure 14 exhibits the mean and "centerline" variations of the Mach number and streamwise component of velocity with distance downstream. The term "centerline" denotes the intersection of the j- and k-midplanes. The centerline Mach profile shows sharp drops due to ramp and reflected shocks and a more gradual decline in the diffuser, where as noted earlier, the flow is highly three-dimensional. In the isolator/combustor shock-train, fairly rapid variations are evident, consistent with the crossing pattern of Fig. 12 where shocks and expansions interleave. The flow is subsequently allowed to over-expand in the nozzle. The variation of mean Mach number is more gradual than the centerline value and is generally lower because of the low speed fluid in boundary layers and separated region (Fig. 13). The various features of the device configuration are also reflected as gradients in the streamwise velocity. However, the change in velocity through the device, both in centerline and mean values, is relatively small — the dominant factor in the reduction of Mach number is the increase in sound speed (temperature). At the horizontal diffuser exit for example, the velocity has been reduced by only about 10% of its freestream value but the Mach number is nearly halved. In the isolator/combustor, the local centerline velocity shows undulations similar to those in Mach number. Further downstream, although the centerline velocity approximately recovers its freestream value, the mean velocity is lower in this flow-through configuration without combustion.

4.2.3 MGD parameters

In order to exert MGD control, the flow in the generator and the accelerator are assumed to be electrically conducting. For the present purpose, a heuristic method is adopted to specify \( \sigma \), consistent with a non-equilibrium ionization method such as noted in Ref. 1. Figure 15 exhibits \( \sigma \) on a \( j \) plane in \( 3-D \) (15(a)) and in normal projection (15(b)). The distributions in the generator and accelerator are obtained independently as products of three \( \text{MGs} \) each. The streamwise extent of the generator region is ensured with \( s \) being streamwise distance, \( s_0 = 7.3481 \) (center of the generator), \( \epsilon = 1 \times 10^{-10} \) and \( \delta = 2.75 \). Parameters for the accelerator are \( s_0 = 14.65, \epsilon = 1 \times 10^{-10} \) and \( \delta = 2.5 \). In the \( y \) and \( z \) directions, the value of \( s_0 \) is specified to be the center of the channel at the corresponding streamwise location, \( \delta \) is the half-width and \( \epsilon = 0.3 \) for the \( z \) direction and 0.1 for the \( y \) direction. The resulting profile at a section of the generator is shown in Fig. 15(c). Thus, \( \sigma \) rises in the boundary layer region from 0.3 at the electrode to its full normalized
Fig. 14 Streamwise variation of u-velocity and Mach number. cl=centerline
value of unity, while on the upper and lower walls, the minimum value is 0.1.

The imposed magnetic field is directed along the y-axis and exhibits the streamwise variation shown in Fig. 16. The normalized value remains one until after the diffuser at which point it diminishes linearly to 0.1. This reduction of field magnitude in the accelerator was dictated by an observed numerical instability, which mandates extremely small time-step-size. Since the present exercise is preliminary and is geared towards exploring problem setup and trends of the MGD-bypass concept, the simple approach was taken to reduce the interaction strength in the generator. The interaction parameter Q is however constant in the entire domain, an equivalent effect may be obtained by reducing the magnitude of the magnetic field as shown. This reduces the local interaction parameter in the accelerator to 0.01 its nominal value in the generator. Note that since the source term is zero wherever \( \phi = 0 \), as a numerical convenience, no attempt is made to restrict \( B \) to conducting regions.

The interaction parameter, \( Q \) is set to 1. Assuming the length scale corresponding to the height/width of the inlet to be 0.6m, the chosen flow parameters correspond to \( \alpha B^2 \sim 1.7pU \) where \( \rho \) and \( U \) denote values local to the generator or accelerator. Representative values of \( \rho U \) for both regions determined from the computed non-MGD solution may be taken to be \( 4\rho_{ref}U_{ref} \) (a single value for the accelerator is difficult to specify because of the rapid variation). Thus, for the conditions above, and assuming a 0.6m \( \times \) 0.6m inlet cross-section area, an interaction parameter \( Q = 1 \) corresponds to \( \alpha B^2 \sim 250T^2/mho/m \).

Figure 14 was also employed to determine potentials on electrodes. The load factor, \( K = -E/uB \) in the generator was set to 0.8. Scrutiny of the mean velocity, Fig. 14, shows that it varies relatively little in the diffuser, and a value of 0.86 may be considered representative. Since a unit magnetic field is imposed in the generator, the nominal imposed \( E \) field is estab-
lished at 0.688. It is then a simple matter to specify voltages, shown in Fig. 10, by factoring in the variable spanwise distance in the diffuser. The decrease in potential with streamwise distance is proportional to the reduction in cross-sectional area in the diffuser. A similar approach is employed to specify potentials in the accelerator, under the assumption of the same mean velocity (Fig. 14) but a load factor of 1.2 (the direction of the electric field is the same as in the generator but is larger than the mean motional emf). Values of potential are however lower because of the reduction in magnetic field strength and increase with streamwise distance in a manner proportional to the distance between electrodes. The effect of the MGD interaction is to reduce (increase) $v$ in the generator (accelerator). Since the computation does not model the external load, but rather maintains a constant potential on the electrodes, the effect is to direct the load factor towards unity. Although not incorporated in this work, it is straightforward to adjust the potential as the solution evolves to maintain the specified load factor.

A semi-coupled strategy is adopted to evolve the solution with MGD interaction. Thus the electric field is first relaxed with the converged flow field without MGD. The former is then frozen while the latter is marched in the usual fashion. Results presented below were obtained with only one iteration of this procedure. Although additional cycles of the procedure are ongoing, the impact of the motional terms on the potential is relatively small as described below and it is thus reasonable to assume that the solution is nearly converged.

4.2.4 Impact of MGD control on flow

Values near the center of the channel may be employed to quantify the effect of magnetic interaction on the bulk flow. To this end, Fig. 17(a) shows Mach number and streamwise velocity along the centerline with and without MGD effects. At the trailing edge of the generator, the MGD interaction (recall that the interaction parameter is set to 1) reduces the streamwise velocity of the flow by 0.1 and the Mach number by about 0.8. These differences persist to the entrance of the nozzle where the difference in values with and without MGD are about the same as at the end of the generator (even though the magnetic field is not zero in the isolator, the electrical conductivity is, and there is no local MGD interaction). Close scrutiny of the expansion in the nozzle suggest that increases in velocity in the initial region are more rapid with MGD. Since the interaction parameter is 0.01 in this region, it is not clear whether this stems from MGD interaction or arises from differences in conditions at the entrance to the nozzle. In any event, neither velocity nor Mach number exceed even match the no-MGD case. Downstream of the accelerator, the rise in velocity and Mach number is much slower. Whereas
the velocity in the no-MGD case nearly recovers its freestream limit, the MGD case shows a lower value, the difference possibly being related to the retarding force in the generator. Current effort is focused on estimating the effect of the accelerator by computing the case with generator on (so the flow at the entrance of the nozzle is the same) but accelerator off.

The effect of MGD interaction on centerline pressure and temperature is shown in Fig. 17(b) and (c) respectively. There is no impact on flow parameters upstream of the generator i.e., its upstream influence is negligible. Both pressure and temperature increase as a result of magnetic effects and at the entrance to the isolator duct, these values are higher with MGD. The subsequent undulations are more striking with magnetic effects: note the high third peak compared to the no-MGD case. Although the pressure is not greatly affected in the accelerator, the temperature is seen to be slightly higher for the MGD case. The higher local sound speed and lower velocity in this region is consistent with the lower Mach number observed in Fig. 17(a).

The response of the flow to MGD forces is summarized in Fig. 18. Fig. 18(a), depicts static pressure on the j mid-plane while 18(b) and (c) show Mach contours near the end of the diffuser and the entrance to the nozzle respectively. Several quantitative effects are evident, when compared with the equivalent results for the no-MGD case (Fig. 12 and 13). Pressure levels are generally higher in the generator and post-generator regions, while Mach numbers are lower as desired. Despite these quantitative differences, the arch-like low Mach number regions are only slightly larger. Evidently, the flowfield structure is not qualitatively affected by the magnetic interaction and the features observed previously in Fig. 13 persist.

4.2.5 Electrical and current fields

Aspects of the electric potential and field in the generator are shown in Fig. 19. Figure 19(a) depicts a 3-D perspective while 19(b) and 19(c) show these variables on two i-planes, the former located at the midpoint of the second electrode and the latter, of smaller cross-section and higher aspect ratio due to the horizontal diffuser, at the insulator between the third and fourth electrodes. The potential varies monotonically over nearly all the region between the anode and cathode. At the electrode plane, the electric field is relatively uniform near the center of the channel but a significant gradient is observed near the wall as a result of high gradients in \( \sigma \) and \( \vec{U} \). The field is generally directed from right to left (looking downstream) and is three-dimensional although the streamwise component is small. The net E-field is opposed to \( \vec{U} \times \vec{B} \), which is dominated by the \( uB_y \) term. The situation at the downstream plane plotted (Fig. 19(c)) is different, partly because of the fact that the sidewalls at
Fig. 18 Aspects of flow field structure with MGD control

this section are insulators and partly because of the downstream evolution of the flow field. Although the electric field has a similar structure, higher perturbations are observed in regions of velocity gradients associated with separation which grows as the cutting plane is moved downstream (Fig. 13). Figure 19(d) depicts the potential and electric fields along a spanwise cut at the vertical center of the two $i$-planes. At the electrode, $x = 6.85$, in the absence of a sheath, the electric field is nearly equal to the design value over much of the domain, suggesting that the $\nabla \cdot (\vec{U} \times \vec{B})$ term in Eqn. 6 is small. At the downstream (insulator) location, the electric field is higher and, since it is opposed to the motional emf, gives rise to a smaller current as described below. An interesting aspect is that the electric field changes direction at the electrode (see electric field vectors on the anode wall in Fig. 19(a)) – this is necessary for current continuity

Fig. 19 Aspects of potential and electric fields in generator
because it is compensated by corresponding changes in electric and velocity fields to satisfy the constraint of current continuity. The current field near the trailing edge of the generator is considerably more complex. At $x = 8.33$, the sidewalls are insulators and current magnitude is small as expected. However, the three-dimensional effects observed in the perspective view is clearly manifested as a constriction in direct current paths to the center of the channel (note that the lines plotted in Fig. 21(b) and (c) are restricted to the plane, whereas those in Fig. 21(a) are not). At the upper and lower parts however, a spirally wound current path is observed, which is correlated with the separation regions of low Mach number shown in Fig. 18. The net result is a region of reversed current relative to the bulk, even though the streamwise velocity is always positive. The phenomenon is consistent with the fact that the electric field (Fig. 19) is relatively uniform and in regions of low velocity overcomes the motional emf to yield locally reversed current.

Aspects of current in the accelerator are shown in Fig. 22. Note that in the 3-D perspective, only one anode is depicted for clarity. Here the current is in the direction of the imposed electric field. Again, the current is relatively uniform at the entrance to the nozzle and magnitudes are much lower than in the generator, because of the small imposed interaction parameter. At the downstream section however, the current paths are considerably curved and inclined upstream. Since the electric field (Fig. 21) is relatively uniform, this is primarily the effect of the velocity field which flares outward as it expands in the nozzle.

The correlation between electrical current and velocity is further elucidated in Fig. 23, which depicts quantities along a line joining the lower and upper walls at the center of the channel at the trailing edge of the diffuser ($x = 8.42$). Fig. 23(a), depicting the $z$-component of the current, shows the previously mentioned reversal near the walls in the separated regions. The velocity profiles, Fig. 23(b) exhibit a core that extends, at this spanwise station, only about a third of the channel, with significant perturbations near both walls. Note again that despite flow separation, the velocity does not exhibit a reversed flow region. The reduction in core velocity is evident. However, the influence of MGD near the walls is relatively minor for reasons to be outlined shortly. The Mach number profile, Fig. 23(c), is similar to the velocity profile though the reduction in core values are larger, because of the heating effect. The Mach number in the arch-like separated structures remains supersonic except for the no-slip dominated region immediately adjacent to the walls.

4.2.6 Force and energetic effects on flow

Aspects of the Lorentz force are depicted in Fig. 24, on the $k$ mid-plane. Since this is a side view, the
generator cross-sectional area is constant. In most of the channel, the force in the generator is opposed to and decelerates the bulk flow. However, near the top and bottom walls, as noted earlier in the context of Fig. 21, the current is reversed in the arch-region and consequently the flow is actually accelerated in the streamwise direction. The effect of the magnetic field in this region is thus to serve as an accelerator and may thus inhibit further increase of separation after it has occurred. In the nozzle, the Lorentz force is consistent with flow acceleration, but is small and exhibits distortion on the centerline near the entrance to the nozzle. It is important to recognize that these results are valid only for the conductivity variation chosen, particularly its property that it is small near the top and bottom surfaces.

The final aspect explored in this preliminary analysis concerns the energy interactions between the magnetic and fluid fields. In Fig. 25 are plotted the total term in the energy equation, $\dot{E} \cdot \dot{j}$, and its two components, the work interaction, $-\dot{j} \cdot (\ddot{U} \times \vec{B})$ and the Joule heating term, $\ddot{j} \cdot \dot{j}/\sigma$ along the centerline. Small streamwise oscillations arising due to variations in $\vec{E}$ have been smoothed to obtain this figure. The major interaction occurs in the generator because of the higher interaction parameter. The total term is negative here, because the extraction of energy from the mean flow is significantly higher than the Joule heating term and indicates a relatively efficient procedure (see e.g., Ref. 16 for a discussion on efficiency). In the accelerator, both components of $\vec{E} \cdot \dot{j}$ are positive since here the current and $\ddot{U} \times \vec{B}$ are generally opposed. Note however that the Joule heating is the smaller component of the total value. This promising observation must be reevaluated for more realistic conditions in which the interaction parameter is increased to realistic values, and turbulence and Hall-effects are included.

**5 Conclusion**

A numerical study has been initiated to explore the use of non-ideal MGD to control two specific situations encountered in scramjet flows, dealing respectively with shock-on-cowl-lip loading and the MHD-energy bypass concept. This paper describes the baseline
established with the simplified formulation in which the spatially variable electrical conductivity is heuristically specified and both turbulence and Hall-effects are neglected. For the first problem, the Type IV interaction, a radially decaying magnetic field is coupled with two different conductivity variations. The first mitigates the near-wall features of the impinging supersonic jet while second perturbs the primary triple point. Results suggest that both techniques are effective, but only at relatively high magnetic interaction parameters. The second approach is superior in that it modifies the basic fluid dynamics of the problem by yielding a Type III interaction at lower local magnetic field values, but requires establishment of such fields relatively further out from the body surface.

A representative fully 3-D simulated dual-plane scramjet configuration has also been established to explore problem setup and numerical issues as well as to understand and verify trends. Both generator and accelerator are modeled with four segmented electrodes pairs each with fixed voltages determined from the mean velocity in the no-MGD solution. The magnetic field is directed vertically upward, and is reduced in the accelerator to avoid numerical instability. Three-dimensional and viscous effects are both shown to cause significant distortion of the flowfield through separation phenomena. Correct trends are demonstrated in both the generator and accelerator, in terms of fluid dynamic as well as electrical quantities. Although a precise characterization of benefits of MGD awaits further exploration, particularly with reference to energetic aspects, the present results show a promising ratio between reversible and irreversible (Joule heating) components.

Acknowledgments The authors are grateful for AFOSR sponsorship under tasks monitored by Maj. W.
Hilbun, PhD and Dr. J. Schmisseur. The authors also acknowledge highly helpful comments and suggestions from several researchers, including R. Mac Cormack, S. Macheret, U. Mehta, J. Shang, M. Visbal and J. Young. This work was also supported in part by a grant of HPC time from the DoD HPC Shared Resource Centers at ASC, CEWES and NAVO.

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American Institute of Aeronautics and Astronautics Paper 2002-2134