INFLUENCES OF INLET FLUCTUATION OF SEED FRACTION ON THE PERFORMANCE OF CCMHD GENERATOR WITH STRONG MHD INTERACTION

Guofeng Lou*, Kazuya Shimizu† and Yoshihiro Okuno‡
Department of Energy Sciences, Tokyo Institute of Technology,
4259 Nagatsuta, Midori-ku, Yokohama 226-8502 JAPAN,
Tel.Fax: +81-(0)45-924-5668, e-mail: lou@es.titech.ac.jp

Abstract
The influences of inlet fluctuation of the seed fraction on the performance of closed-cycle MHD generator have been investigated with the time dependant two-dimensional numerical simulation. The nonuniformity scale of the seed fraction induced by the inlet fluctuation is comparable to the length of the channel or small than that. It is confirmed that the inlet fluctuation of the seed fraction can cause the instability in the non-equilibrium plasma and deteriorate the performance of the generator. With a constant frequency, increasing the amplitude of inlet fluctuations, the fluctuation of the enthalpy extraction ratio enhances, meanwhile the mean value decreases. With the constant amplitude, varying the frequency of the inlet fluctuation, the fluctuation of the enthalpy extraction ratio becomes larger when the frequency of the inlet fluctuation approaches to 8-10kHz. Although the inlet fluctuation induces the fluctuation of the plasma parameters and the flow field parameters, the two kinds of fluctuations have different characteristics when the inlet fluctuation with the higher frequency. That is, increasing the frequency, the strength of the former keeps approximately constant, however the latter weakens. The inertia of the flow field is employed to explain the reason. For the pulse-like random inlet fluctuation with high dominant frequency, the fluid field cannot catch up with the inlet fluctuation of the seed fraction, and a frequency of 5-6 kHz becomes dominant in the flow field, which can be related to the resident time of the working medium in the channel.

Keywords: fluctuation, Seed fraction, Closed-Cycle MHD generator, Non-equilibrium plasma

Introduction
The closed-cycle (CC) MHD electrical power generation is a kind of direct energy conversion systems. The seeded noble gas is used as the working medium instead of the metal in a conventional power generator. As well be known, the electrical conductivity of the working medium is a critical parameter and the higher value is preferred for the advanced MHD generator performance. Like that of the metal, the electrical conductivity is a function of both of the density of the free charge carriers and their mobility. As the main free charge carries, the electrons with high temperature will be helpful. Because the non-equilibrium plasma is employed as the working medium, on the one hand, it has a relatively high electron temperature for high electrical conductivity and at the same time keeps the heavy gas temperature in the range of the bearing temperature of the facility material. On the other hand, the fluctuation and the nonuniformity are easily induced [1,2]. Two important efficiencies are usually used to estimate the performance of CCMHD generator. One is the enthalpy extraction ratio (E.E.), the other is the isentropic efficiency (I.E.). The nonuniformity and fluctuation of plasma can affect the efficiencies of MHD generator significantly [2]. In the past decades, the reasons, which cause the nonuniformity and fluctuation of the non-equilibrium plasma, have been investigated. Contrasting the much research work on the internal reasons such as the instability of the plasma, the boundary layer and the phenomena near the electrodes [3,4,5], a relatively little work has been done on the “external reasons” such as the input thermal energy and inlet seed fraction of the generator channel [6].

The experimental researches on the disk CCMHD generator for the purpose of the commercial MHD generator have been made at Tokyo Institute of Technology, Japan. The fluctuations of the output power and the parameters were found in both the Fuji-1 MHD
blow-down facility and in the shock-tube driven MHD generation experimental facilities [7,8,9]. In the experiments of the shock-tube driven MHD generation facility, the fluctuation of the flow field that appears as the fluctuation of the static pressure has a frequency about 8 kHz [9,10]. At the same time, the fluctuation of the seed fraction with 10-15 kHz was also found in the experiment, although the fluctuation was measured not at the generator inlet but at the stagnation point of the shock tube end edge [11].

Quasi-one dimensional numerical simulations had been made on the performance of CCMHD generator with the fluctuation of inlet seed fraction [6,12]. In reference 6, the random inlet fluctuation of the seed fraction was checked. It was confirmed that a sufficiently large fluctuation of the inlet seed fraction reduces the output power and isentropic efficiency considerably. In reference 12, the effect of the fluctuation of the inlet seed fraction on the subsonic CCMHD generator was investigated. The transient phenomenon caused by the sudden change of the inlet seed fraction was studied also. As one of the most important parameters of the CCMHD generator, influences of the inlet fluctuation of the seed fraction should be studied more detailedly. For this purpose, the r-z two dimensional numerical simulation has been carried out in the present work.

In the practical facility, the inlet fluctuation of the seed fraction with the relatively small amplitude is very difficult to be excluded. Present work is concern with this kind of inlet fluctuation. The frequencies of the inlet fluctuation of the CCMHD generator cover a very wide range. To determine the frequency range being studied in the present work, a dimensionless parameter $\ast$ can be introduced. $\ast = 1/(f \ast)$, here $f$ is the frequency of the inlet fluctuation and $\ast$ is the residence time of the working medium in the channel ($\ast \leq 250 \mu s$). The larger $\ast$ refers to the smoother distribution of the seed fraction in the channel. In fact the $\ast$ is the ratio of the nonuniformity scale of the seed fraction to the length of the channel. For $\ast > 1$ the inlet fluctuation with a low frequency such as several tens or a hundred Hz, the plasma structure in channel can be treated as a steady state. For this situation the steady analysis with different operating conditions can be used to describe the performance of the CCMHD generator. Present work is concentrated in the inlet fluctuation with $\ast \leq 1$, where the nonuniformity scale is comparable to the channel length or smaller than that.

### Basic equations for simulation

Based on the engineering conditions, charge-neutrality and low magnetic Reynolds number assumptions are hired. A simple two-temperature model is adopted for non-equilibrium plasma. So the governing equations of MHD plasma, which consists of noble gas atoms, noble gas ions, seed atoms, seed ions and electrons, can be written as follows using well-known nomenclatures:

1) **Governing equations of heavy particles:**

   **Continuity equation**
   \[
   \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0
   \]  

   **Momentum equation**
   \[
   \frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla P + \nabla \cdot \tau_{ij} + \vec{j} \times \vec{B}
   \]  

   **Energy equation**
   \[
   \rho \left[ \frac{\partial (C_i T)}{\partial t} + \vec{u} \cdot \nabla (C_i T) \right] = -p \nabla \cdot \vec{u} + \Phi + \nabla \cdot (k \nabla T) + \frac{\vec{j}^2}{\sigma}
   \]  

2) **Governing equations of charged particles:**

   **Continuity equation**
   \[
   \frac{\partial n_i^e}{\partial t} + \nabla \cdot (n_i^e \vec{u}) = \dot{n}_i^e = k_{i} \rho_{i} n_i - k_{i} \rho_{i}^{2} n_i
   \]  

   **General Ohm's Law**
   \[
   \vec{j} + \frac{\vec{B}}{B} \vec{j} \times \vec{B} = \sigma (\vec{E} + \vec{u} \times \vec{B}) = \sigma \vec{E}
   \]  

   **Electron energy equation**
   \[
   \frac{\partial \left[ 2 n^e c^2 \right]}{\partial t} = 3 n_i m_i k_{i} (T_i - T_e) \sum_{s} \frac{v_{eh}}{m_{i}} + \sum_{i} \dot{n}_i^e \left( \frac{3}{2} kT_e + \varepsilon_i \right)
   \]  

3) **Maxwell equations:**

   \[
   \nabla \times \vec{E} = 0
   \]  

   \[
   \nabla \cdot \vec{j} = 0
   \]  

4) **Perfect gas status equation:**

   **American Institute of Aeronautics and Astronautics**
\[ p = \rho RT \] 

**Numerical procedures**

Present calculations are based on the shock tube driven disk shaped MHD generator Disk-1S1 at Tokyo Institute of Technology [9,10]. The cylindrical system of coordinates is adopted. The calculation region is shown in Fig.1, which covers the area from the upstream edge of anodes (r=105 mm) to the downstream edge of cathodes (r=275 mm), and between two walls in the vertical direction. The heights at inlet and outlet are 19.7 mm and 23.5 mm respectively. Magnetic field is applied in the z-direction. The mesh number is 68 in r-direction and 40 in z-direction. The time step is 0.025 \mu s. CIP method is used to solve hyperbolic equations [13]. From the general Ohm's law and Maxwell equations, a potential function two-order elliptic equation is obtained as follows and is solved by the finite difference method:

\[
\begin{align*}
\frac{\partial}{\partial r} \left[ \frac{\sigma r}{1 + B^2} \left( \frac{\partial \phi}{\partial r} - \frac{\beta}{r} \frac{\partial \phi}{\partial \theta} - \beta u_r B - u_\theta B \right) \right] \\
+ \frac{\partial}{\partial \theta} \left[ \sigma \left( \frac{\beta}{1 + B^2} \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial \phi}{\partial \theta} + u_r B - \beta u_\theta B \right) \right] \\
+ \frac{\partial}{\partial z} \left[ \frac{\sigma r}{1 + B^2} \frac{\partial \phi}{\partial z} \right] = 0.
\end{align*}
\] 

(10)

Here \( \phi \) is the potential function and is defined by the Maxwell equation as follows:

\[
E_r = -\frac{\partial \phi}{\partial r}, \quad E_\theta = -\frac{\partial \phi}{r \partial \theta}, \quad E_z = -\frac{\partial \phi}{\partial z}.
\] 

(11)

For a supersonic flow, the inlet parameters are given as inlet boundary conditions. The outlet boundary is decided by free boundary condition. No-slip boundary condition is used for walls. For the elliptic equation the boundary conditions are given for every boundary surface [14,15].

**Calculation conditions**

Table 1 shows the operating conditions of the generator for present simulation, which are based on the experimental conditions. The fluctuation is introduced as the inlet condition, and for seed fraction it is described as follows:

\[ S_f = S_{f0} \times (1 + \text{fluctuation value ratio}) \] 

(12)

where \( S_{f0} \) is the basic seed fraction value. For sinusoidal fluctuation the second term in the parenthesis is \( A \sin(\omega t) \), where \( A \) is the amplitude, \( \omega \) is the angular frequency, and \( t \) is the time.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>The operating conditions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working gas</td>
<td>Ar-Cs</td>
</tr>
<tr>
<td>Inlet stagnation temperature</td>
<td>2500 [K]</td>
</tr>
<tr>
<td>Inlet stagnation pressure</td>
<td>0.25 [MPa]</td>
</tr>
<tr>
<td>Magnetic flux density</td>
<td>2.9-0.3 [T]</td>
</tr>
<tr>
<td>Inlet swirl ratio</td>
<td>0.83</td>
</tr>
<tr>
<td>Thermal input</td>
<td>2.20 [MW]</td>
</tr>
<tr>
<td>Seed fraction</td>
<td>1.2x10^{-4}</td>
</tr>
<tr>
<td>Load resistance</td>
<td>0.140 [\Omega]</td>
</tr>
</tbody>
</table>

**Results and discussion**

**The effect of the sinusoidal inlet fluctuation with different amplitude**

If there is a fluctuation of the seed fraction at the inlet, it will go to downstream with the working medium and cause the temporal fluctuation and spatial nonuniformity of the plasma in the channel. The effect of the fluctuation of the inlet seed fraction has been investigated by various conditions. For the sinusoidal fluctuation, the frequency and the amplitude are the typical parameters to describe the characteristic of the fluctuation. At first the effect of the inlet fluctuation of the seed fraction with different amplitude is discussed. For a constant frequency (10 kHz) and the basic value of seed fraction being 1.2x10^{-3}, the effect of varied amplitude is shown in Fig.2. In this figure, the statistic value of the fluctuation of the enthalpy extraction ratio is drawn as a function of amplitude of the inlet fluctuation. The black square is the time average value of the fluctuation of enthalpy extraction ratio. The upset triangle is the maximum value. The triangle is the minimum value and the error bar is the standard deviation of the fluctuation of the efficiency. It can be found from
this figure that with the increase in the amplitude of inlet fluctuations, the fluctuation of the efficiency is enhanced, meanwhile the mean values decrease. For the isentropic efficiency the same trend was found. To explain the reason, the phenomena caused by the inlet fluctuation of the seed fraction are revealed. For the amplitude of the inlet fluctuation being 10% of the basic value, Fig.3 shows

\[ \text{Fig.2 The enthalpy extraction ratio versus the amplitude of the inlet fluctuation (1.2 \times 10^3, 10 kHz).} \]

\[ \text{Fig.3 The distribution of the electron temperature in main flow for 4 typical moments of one period (10%, 10 kHz).} \]

\[ \text{Fig.4 The distribution of the electrical conductivity in the r-z plane (10%, 10 kHz).} \]

\[ \text{Fig.5 The current density vector in the r-z plane (10%, 10 kHz).} \]

\[ \text{Fig.6 The distribution of the static pressure in main flow of 4 typical moments of one period (10%, 10 kHz).} \]

the distributions of the electron temperature in the main flow along the radius for 4 typical moments in one period. **The time interval between these 4 moments is the quadrant of the period. In this figure the wider solid curve denotes the distribution of the electron temperature without inlet fluctuation. It can be seen in Fig.3 that the electron temperature is about 4100 K and the distribution is smooth for that without inlet fluctuation. For the presence of the inlet fluctuation, the fluctuation of the electron temperature is large. The fluctuation mainly covers the range from 3000 K to 6000K. The distribution of the electrical conductivity in the r-z plane is drawn in Fig.4. In this figure the nonuniformity of the electrical conductivity can be found in both the r-direction and the z-direction. Since there is the nonuniformity of the electrical conductivity in the channel, with the Hall effect, the eddy current will be induced. Figure 5 shows the current density vector in r-z plane. From this figure the eddy current caused by the fluctuation of the seed fraction can be seen clearly. Thus, this will deteriorate the performance of the generator. From the governing equation of the heavy particle, it can be seen that the electromagnetic field will affect the flow field by the Lorentz force term in the momentum equation and by the Joule heating term in the energy equation. The distribution
of the static pressure for 4 typical moments in one period is plotted along the radius in Fig.6. The periodic response of the flow field to the fluctuation of the seed fraction can be found in this figure. It shows that the inlet fluctuation of the seed fraction not only causes the fluctuation of the plasma parameters but also the fluctuation of the flow field, meanwhile the fluctuation moves to the downstream.

![Effective electrical conductivity](image)

Fig.7 The time trace of the effective electrical conductivity for the part of the main flow ($1.2 \times 10^3$).

Increasing the amplitude of the inlet fluctuation will cause the larger nonuniformity in the channel. The conception of the effective electrical conductivity is used to estimate the nonuniformity quantitatively. The effective electrical conductivity is calculated here for the part of the main flow (from 0.145 to 0.22 m) by the following equation:

$$
\sigma_{eff} = \frac{\langle j^2 \rangle_{av}}{\langle j \cdot \overline{\varepsilon} \rangle_{av}}
$$

(13)

The selection of the part of the main flow in the channel is to exclude the effect of the electrodes. Figure 7 shows the time trace of the effective electrical conductivity. The fluctuation of the effective electrical conductivity results from the change of the distribution of the seed fraction. The effects of the nonuniformity caused by the inlet fluctuations with different amplitude can be seen. The larger amplitude of inlet fluctuation causes the stronger fluctuation of the effective electrical conductivity and decreases the time average value. This will result in the larger fluctuation of the efficiency and deteriorate the performance more. It should be noted that the fluctuation of the effective electrical conductivity for 1% in Fig.7 is quite small comparing with other curves. That is because there is no occurrence of the ionization instability, in contrast with others.

![Enthalpy extraction ratio](image)

Fig.8 The enthalpy extraction ratio versus the frequency of the inlet fluctuation ($1.2 \times 10^3$, 5%).

**The effect of the sinusoidal inlet fluctuation with different frequency**

In this section, the effect of the inlet fluctuation of the seed fraction with different frequency is discussed. For the basic value of the seed fraction being $1.2 \times 10^3$, with the invariant amplitude (5%), the statistic value of the fluctuation of the enthalpy extraction ratio is drawn versus the frequency of the inlet fluctuation in Fig.8. It can be seen that the fluctuation of enthalpy extraction ratio becomes larger when the frequency of inlet fluctuation approaches to 8-10kHz, and the larger fluctuations of the efficiency have the smaller mean values. The same situation was found for the isentropic efficiency. To explain the reason of this phenomenon, the distributions of the plasma parameters are plotted in Fig.9 and Fig.10. Figure 9 shows the representative distribution of the electron temperature in the main flow along radius for different frequencies of the inlet fluctuation. The electron temperature is normalized for comparing each other and the distributions can represent the ranges of the fluctuation for different frequency respectively. The wider solid curve with the 3 kHz inlet fluctuation is quite different from other curves. It changes smoothly and has the smaller range of the fluctuation. When the frequency of the inlet fluctuation is equal to 5 kHz, the range of the fluctuation becomes larger and there are acute changes for the electron temperature in the channel, then, with the increase of the frequency the range of fluctuation is approximately constant (the weaken is small). The similar situation can be found in Fig.10 for the electron number density. In two figures
when the frequency of the inlet fluctuation being 5 kHz, the sudden changes for the range of the plasma fluctuation are caused by the instability of the plasma. Figure 11 shows the normalized distribution of the static pressure for the same conditions as for the Fig.9 and Fig.10. In Fig.11, as shown in the previous two figures, when the frequency of inlet fluctuation increases from 3 to 10 kHz the fluctuation of the static pressure enhances. However, when the frequency increases further, unlike the electron temperature and the electron number density, the fluctuation of the static pressure becomes small. The enlarging of the static pressure fluctuation for the frequency increasing from 3 to 10 kHz is because of the enhancing of the fluctuation of the plasma parameters. To explain the weakening of the fluctuation of the static pressure when the frequency increases to the larger value, the fluctuation of the parameters at the same position of the channel (r = 0.22 m, in main flow) is checked. Figure 12 shows the fluctuations of the seed fraction, the electron number density and the static pressure at the same position. Comparing the times τ₁, τ₂ and τ₃ in this figure, it is found that there are time delays between the variations of the different parameters. This is because the one order time derivative term in the governing equations of the heavy particle, and in the continuum equation of the charged particle. The difference of the τ₂ and τ₃ means that the inertias for the electron number density and the static pressure are different to follow the change of the seed fraction. Because having the larger inertia, the fluctuation of the static pressure for high frequency is suppressed. Although the time delay will decrease with the increase of the frequency of the seed fraction fluctuation, the decreasing is slow comparing with the increasing of the frequency. For the electron number density the inertia is relatively small, so for the high frequency (in present work) the inertia’s effect is not so significant. By now it is clear that the fluctuation of the efficiency being enhanced from 1 kHz to 10 kHz is because the plasma instability occurs and the nonuniformity will increase with the increase in the frequency even for the same amplitude of the plasma fluctuation (there are more convexes and concaves in the channel at one moment). The weakening of the fluctuation of the efficiency above 10 kHz is because the fluctuation of the flow field is suppressed gradually with the increase of the frequency.

Fig.9 The typical distribution of the electron temperature along radius for the inlet fluctuation with different frequency.

Fig.10 The typical distribution of the electron number density along radius for the inlet fluctuation with different frequency.

Fig.11 The typical distribution of the static pressure along radius for the inlet fluctuation with different frequency.
The fluctuations of the parameters at the position $r = 0.22$ (m) in the main flow.

Fig. 12

The enthalpy extraction ratio versus the value range of the inlet fluctuation.

Fig. 13

The enthalpy extraction ratio versus the time interval of the inlet fluctuation.

Fig. 14

The effect of the pulse-like random inlet fluctuation

The effect of the pulse-like inlet fluctuation of the seed fraction on the performance of the generator is also investigated. The pulse-like inlet fluctuation, where the pulse value for each time interval is random, is added at the inlet of the channel. For the basic value of seed fraction being $1.2 \times 10^{-3}$, the time interval and the random value range (from minimum to maximum) are changed respectively. The statistic values of the fluctuation of the enthalpy extraction ratio are shown in Fig. 13 and Fig. 14. With the constant time interval of the inlet fluctuation (33 $\mu$s), it can be seen from Fig. 13 that increasing the random value range enlarges the fluctuation of the enthalpy extraction ratio and the mean value decreases. With the constant random value range of the inlet fluctuation (3%) and changing the time interval only, Fig. 14 shows that the fluctuation of the enthalpy extraction ratio enhances when the time interval approaches to 30 $\mu$s. These phenomena are similar to that for the sinusoidal inlet fluctuation and the same explanation can be employed. For the same set of the inlet fluctuation shown in Fig. 14, the Fast Fourier Transform (FFT) method is used to find out the dominant frequency of the inlet fluctuation and the fluctuation of the static pressure at the radius being 0.145 (m) in the main flow of the channel. For every time interval value, it produces a pair of the dominant frequencies of the inlet fluctuation of the seed fraction and the fluctuation of the static pressure in the channel. Those are drawn as a function of the time interval of the inlet fluctuation in Fig. 15. It is clear that there is difference between the frequency of the inlet seed fraction fluctuation and the frequency of the static pressure when the time interval is short, and then the dominant frequency of the static pressure is about 5-6 kHz. Phenomenally, it seems that the fluid flow cannot respond to the inlet fluctuation of the seed fraction with high frequency by the same frequency.

Fig. 15

The Dominant frequencies versus the time interval of the inlet fluctuation.

The pulse-like random inlet fluctuation of the seed fraction includes a lot of components with different frequencies. When the time interval of the random
fluctuation is long and the dominant frequency of the inlet seed fraction fluctuation is below about 10kHz, the fluid flow can respond to the fluctuation of seed fraction with the low dominant frequency. On the other hand, for the dominant frequency of inlet seed fraction fluctuation above 10 kHz, because the effect of the inertia of fluid flow appears relatively, the high frequency component in the fluid flow becomes weak, although the component remains. Then the low frequency component becomes dominant in the flow field. It should be noted here that in the Hall type generator with one load resistance such as a disk shaped generator, unlike the segmented Faraday type generator, the plasma and fluid properties in the channel can be affected not only locally but also under the regime as a whole of the channel. Thus, the residence time of fluid flow in the channel is one of the characteristic times. Under the present conditions, the residence time is about 250± s, which corresponds to a frequency of 4kHz and it is almost the same as the dominant frequency of the static pressure of 5-6kHz mentioned above.

**Conclusion**

The influences of the inlet fluctuation of the seed fraction on the performance of CCMHD generator have been investigated by the time dependent r-z two-dimensional numerical simulation. For the nonuniformity scale of the seed fraction being comparable to the length of the channel or smaller than that, the mechanism of the effect has been discussed. The main conclusions are summarized as follows.

For sinusoidal inlet fluctuation of the seed fraction, with the constant frequency, increasing the amplitude of inlet fluctuations, the fluctuation of the enthalpy extraction ratio enhances, meanwhile the mean value decreases.

With the constant amplitude, varying the frequency of the inlet fluctuation, the fluctuation of the enthalpy extraction ratio becomes stronger when the frequency of the inlet fluctuation approaches to 8-10kHz, and the mean value become small. Increasing the frequency further, the fluctuation of the flow field caused by the inlet fluctuation of the seed fraction is suppressed gradually.

For the pulse-like random inlet fluctuation of the seed fraction, the same tendency as that for sinusoidal inlet fluctuation was found for the constant random value range condition and the constant time interval condition respectively. For the pulse-like random inlet fluctuation with high dominant frequency, the fluid field cannot catch up with the inlet fluctuation of the seed fraction, and the dominant frequency is about 5-6 kHz.

**Reference**


8 – 9

American Institute of Aeronautics and Astronautics


