Preliminary Analysis of MHD Generator Using Coal Synthesized and Preheated Gas Combustion

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Abstract
If we could use the coal synthesized and preheated gas (CO+H₂) as the fuel of MHD generator, the combustion plasma has the gross electrical conductivity. Because of the plasma is high. Also, the conductivity is high in boundary layers, as the gas has no slag. Therefore it is necessary for electrode to keep high temperature keeping conductivity in boundary layers high. From this characters, we expected that we design an efficient MHD generator with the coal synthesized and preheated gas plasma. The analysis of MHD generator with this plasma is a few. In this study, the 3D characteristics in a 500 MWth Faraday channel designed for coal synthesized and preheated gas combustion plasmas were analyzed on the basis of electrical equations coupled with those of a set of palabolized MHD flow equations. To decide the MHD channel geometry was carried out by Q1D computer code. The result of this, the output of generator was 89.92MW. 3D analysis was carried out for this channel. The result of 3D analysis, it was clearly to rise 3D phenomena in the channel.

Introduction
Volume distribution of gasified coal (CO+H₂) is 2:1. In order to achieve the plasma with high conductivity, gas will be burn up with oxygen and preheated to use waste heat. Also gas has no slag, thus hot electrode is needed to keep conductivity high near the electrode. For those conditions, it will be expected that conductivity and interaction in the channel will be very high and strong. Meanwhile, MHD generator with gasified coal combustion has possibility to be toppor in system of CO₂ separation and resumption system. Thus it is important to analyze MHD generator like this. In terms of this reasons, we Preliminary analyzed MHD generator using fuel that is CO, H₂, O₂ and seed preheated waste heat of MHD generator at 1050K and combustion.

Quasi one-dimensional design
To decide channel geometry that thermal input is 500MW, Q1D computer code was carried out. This code supposes the next points,

1. Plasma is electrical neutral, magnetic Reynolds number is low and the ion slip doesn’t exist.
2. Friction and heat loss are valued by flat plane turbulence model that is considered by plane roughness.
3. consider boundary layers thickness to progress, distribution of temperature in boundary layers were given by G factor. G factor was given at each cross section.
4. voltage drop near the electrode is not considered.
5. reek resistance was not considered.

From these assumptions, Q1D code solves next equations

\[ p = \rho RT \] (1)
\[ p\mu A = m_0 \] (2)

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\[ m\frac{du}{dx} + A\frac{dp}{dx} - AJ_B - f = 0 \]  
(3)

\[ m\frac{dh}{dx} + A\frac{du}{dx} - (J_1E_x + J_2E_y)A = q \]  
(4)

\[ \nabla \times E = 0 \quad (5), \nabla \cdot J = 0 \quad (6) \]

\[ J = \sigma(E + u \times B) - \frac{\beta}{|B|} (J \times B) \]  
(7)

G factor was given by: \(^2\)

\[ G = (\sigma)^{\left(1 + \frac{\beta^2}{\sigma}\right)} - (\beta)^{\frac{2}{2}} \]  
(8)

in this study, distribution of temperature in boundary layers were given by 1/7 powers low. Pressure is constant in the cross section. Boundary layers thickness\(^3\) is given by

\[ \delta = 0.37 \left(\frac{\mu x}{V}\right)^{-0.2} \]  
(9)

where \(x' = x + 1\).

The thermodynamical properties of working gas were given by the function made by result of calculation. Calculation is carried out by computer code that was developed by Electrotechnical Laboratory\(^4\). This code can calculate the thermodynamical properties which is enthalpy, density, conductivity, and so on.

In this study, the working gas that mole fraction was CO: H\(_2\)=2:1, equivalence ratio of oxygen is 1, seed is potassium was 1wt%. thermodynamical properties of working gas were given by

\[ Q(p,T) = \sum_{\psi_0} \left( \frac{1}{\sum_{\psi_0} C_1 \mu^{-2} T'} \right) \]  
(10)

This equation gives thermodynamical properties in some range. pressure range is 0.05MPa–2MPa, temperature range is 2000K–3800K.

The fixed conditions to design channel is below.

1. Thermal input is 500MW
2. Channel length is 10m
3. Cross section geometry is square
4. temperature is 3200K at inlet
5. velocity gradient\(\frac{du}{dx}\) is constant along the channel
6. distribution of magnetic field is 4T at inlet, increase proportional from inlet to 0.8m, is 5T from 0.8m to 9.2m, decrease proportional from 9.2m to outlet, is 4T at outlet.

Pressure at inlet, Mach number at inlet, velocity gradient and load factor was changed and to design the channel is repeated. Requirement of channel design sets stagnation pressure at outlet was near 0.1MPa. the result of Q1D designed parameters and geometry were shown table1 and Fig. 1 respectively.

<table>
<thead>
<tr>
<th></th>
<th>Inlet</th>
<th>Outlet</th>
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<tbody>
<tr>
<td>Height &amp; width</td>
<td>0.302m</td>
<td>0.825m</td>
</tr>
<tr>
<td>Temperature</td>
<td>3200K</td>
<td>2709K</td>
</tr>
<tr>
<td>Pressure</td>
<td>0.64MPa</td>
<td>0.064MPa</td>
</tr>
<tr>
<td>Velocity</td>
<td>911.7m/s</td>
<td>791m/s</td>
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<tr>
<td>Mach number</td>
<td>0.940</td>
<td>0.938</td>
</tr>
<tr>
<td>Wall Temperature</td>
<td>2000K</td>
<td></td>
</tr>
<tr>
<td>Velocity gradient</td>
<td>-12m/s/m</td>
<td></td>
</tr>
<tr>
<td>Mass flow</td>
<td>59.43</td>
<td></td>
</tr>
<tr>
<td>Load factor</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Thermal input</td>
<td>500MW</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>89.92MW</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 parameters of designed channel

![Q1D result (channel geometry)](image)

Fig. 1 designed channel geometry

In Fig. 1, line shows channel height and width, broken line shows boundary layer.

From Table1, enthalpy efficiency was 17.98%. Fig. 2 shows faraday current density and hall electric field along the channel. From Fig. 2, faraday density was over 1x10\(^3\) A/m\(^2\) at wide range of channel.
Thermal analysis

Three-dimensional analysis was carried out by modified computer code which was developed by Argonne national laboratory. This code solves parabolized MHD flow equations. Parabolic approximation was used by solution of flat plate flow problems. It is consist of neglecting the diffusional fluxes in the axial direction and considering the pressure gradient in the axial momentum equation to be uniform over the duct cross section. The parabolic approximation doesn’t introduce reverse flow. The equations solved were below

Mass continuity

\[ \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0 \]  

(11)

x-momentum

\[ \frac{\partial}{\partial x} (\rho u u) + \frac{\partial}{\partial y} (\rho u v) + \frac{\partial}{\partial z} (\rho u w) = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} (\tau_{xy}) + \frac{\partial}{\partial z} (\tau_{xz}) + J \beta \]  

(12)

y-momentum

\[ \frac{\partial}{\partial x} (\rho u v) + \frac{\partial}{\partial y} (\rho v v) + \frac{\partial}{\partial z} (\rho v w) = - \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} (\tau_{xy}) + \frac{\partial}{\partial z} (\tau_{yz}) - J \beta \]  

(13)

z-momentum

\[ \frac{\partial}{\partial x} (\rho u w) + \frac{\partial}{\partial y} (\rho v w) + \frac{\partial}{\partial z} (\rho w w) = - \frac{\partial p}{\partial z} + \frac{\partial}{\partial y} (\tau_{zx}) + \frac{\partial}{\partial z} (\tau_{zz}) \]  

(14)

enthalpy

\[ \frac{\partial}{\partial x} (\rho u h) + \frac{\partial}{\partial y} (\rho v h) + \frac{\partial}{\partial z} (\rho w h) = - \frac{\partial q_x}{\partial y} + \frac{\partial q_z}{\partial y} + u \frac{\partial}{\partial x} (\tau_{xy}) + v \frac{\partial}{\partial y} (\tau_{yz}) + w \frac{\partial}{\partial z} (\tau_{zz}) + J^2 + D \]  

(15)

where

\[ D = \mu \left[ 2 \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 \right] \]

state equation.

\[ p = \rho RT \]  

(16)

Because the flow velocities are high in the channel, it is necessary to consider the effect of turbulence. In this study, the product of turbulent viscosity and the gradients of flow variable represented the turbulent fluxes. The turbulent viscosity was calculated from the local values of the turbulent kinetic energy, \( k \), and its dissipation rate, \( \varepsilon \), from below.

\[ \mu_t = C_{\mu} \frac{k^2}{\varepsilon} \]  

(17)

the turbulent fluxes are calculated below

\[ \tau_{ij} = (\mu_t + \mu_0) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  

(18)

\[ q_i = \left( \frac{\mu_t}{\sigma_h} + \mu_0 \right) \left( \frac{\partial h}{\partial x_j} \right) \]  

(19)

the values of \( k \) and \( \varepsilon \) are obtained from the solution of the following transport equations

kinetic energy of turbulence

\[ \frac{\partial}{\partial t} (\rho u k) + \frac{\partial}{\partial x} (\rho u v k) + \frac{\partial}{\partial y} (\rho v w k) + \frac{\partial}{\partial z} (\rho w w k) = \frac{\partial}{\partial x} \left( \mu_t \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_t \frac{\partial k}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_t \frac{\partial k}{\partial z} \right) + G - \rho \varepsilon \]  

(20)

dissipation rate

\[ \frac{\partial}{\partial t} (\rho \varepsilon) + \frac{\partial}{\partial x} (\rho u \varepsilon) + \frac{\partial}{\partial y} (\rho v \varepsilon) + \frac{\partial}{\partial z} (\rho w \varepsilon) \]  

\[ = \frac{\partial}{\partial x} \left( \mu_t \frac{\partial \varepsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_t \frac{\partial \varepsilon}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_t \frac{\partial \varepsilon}{\partial z} \right) + \frac{C}{k} - C \frac{\rho \varepsilon^2}{k} \]  

(21)

where \( G \) represents the production of the kinetic energy of turbulence due to the interaction of the shear stresses with the velocity gradients.

Electrical fields and currents were coupled under
the infinite-segmentation approximation. In this approximation, the axial variations of flow and electrical variables were neglected in comparison with their cross plane variations. The Maxwell equations and the ohm's law were first written under the MHD approximation as

\[ \nabla \times E = 0 \quad \nabla \cdot J = 0 \]  

\[ J = \sigma (E + U \times B) - \frac{\beta J \times B}{|B|} \]  

(22)

(23)

in this study, magnetic field is \( B = (0,0,B) \), directed along \( z \) direction. Under the infinite-segmentation approximation,

\[ \frac{\partial E}{\partial x} = \frac{\partial J}{\partial x} = 0 \]  

(24)

from this equations, reduce the current continuity equations

\[ \frac{\partial J_x}{\partial y} + \frac{\partial J_z}{\partial z} = 0 \]  

(25)

expressing \( E = -\nabla \phi \), and using the infinite-segmentation approximation, it can be shown that for Faraday channels the potential \( \phi \) may be expressed as

\[ \phi(x,y,z) = -E_x + \phi^*(y,z) \]  

(26)

where \( E_x \) is a function \( x \) only. Substitution of the expression for currents in terms of \( \phi^* \) in the current continuity equations gives the following equations for \( \phi^* \)

\[ \frac{\partial}{\partial y} \left( \sigma \frac{\partial \phi^*}{\partial y} \right) + \frac{\partial}{\partial z} \left( \sigma \frac{\partial \phi^*}{\partial z} \right) = E_x \frac{\partial}{\partial y} \left( \frac{\sigma \beta}{1 + \beta^2} \right) + B \frac{\partial}{\partial y} \left( \frac{\sigma \beta}{1 + \beta^2} (\beta y - u) \right) \]  

(27)

Conditions of 3D-analysis

A Condition of 3D 3D-analysis was used parameters that were decided by Q1D design. But 3D-analysis code has a distribution at inlet. The distribution is given by 1/7 power low in boundary layers. Therefore, designed parameters were used condition at channel center of inlet. The conditions of 3D-analysis shows Table2. In table2, width and height are different from Q1D design. Because we used geometry designed by Q1D code, reverse flow occurred. Since different geometry was used in 3D-analysis. The channel analyzed had as same volume as Q1D designed and width and height were shown in Fig. 3

<table>
<thead>
<tr>
<th>Temperature at center</th>
<th>3200K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>0.64MPa</td>
</tr>
<tr>
<td>Velocity at center</td>
<td>911.7m/s</td>
</tr>
<tr>
<td>Mach number at center</td>
<td>0.940</td>
</tr>
<tr>
<td>Mass flow</td>
<td>57.7kg/s</td>
</tr>
<tr>
<td>Thermal input</td>
<td>463MW</td>
</tr>
</tbody>
</table>

Table2 Condition of 3D-analysis

In this analysis, the channel geometry was different from design so properties in the channel were different from the properties of Q1D design. So the conditions of load factors were changed to want to analyze the channel that has more nearly properties than the properties of Q1D design. The conditions of load factor analyzed were tow ways. One was constant another was changed.

Result of 3D-analysis

![Graph showing channel geometry analyzed](image)

Fig3 channel geometry analyzed

The overviews of result were shown Table3. (A1 is constant load factor, A2 is changed load factor) In this table, output is lower than Q1D design because velocity gradient along the

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>2991K</td>
<td>2852K</td>
</tr>
<tr>
<td>Pressure</td>
<td>0.478MPa</td>
<td>0.205MPa</td>
</tr>
<tr>
<td>Velocity</td>
<td>247.2m/s</td>
<td>540.9m/s</td>
</tr>
<tr>
<td>Out put</td>
<td>23.43MW</td>
<td>44.64MW</td>
</tr>
</tbody>
</table>

Table3 parameters at outlet of 3D-analysis
Fig 4: Axial velocity along the channel.

Fig 5: Pressure along the channel.

Fig 6: Average Faraday current in each cross section.

Fig 7: Average conductivity in each cross section.

Fig 8: Hall electric field in each cross section.

The channel was bigger than Q1D design. Different geometry and different pressure gradient along the channel caused the fact. Fig. 4 shows axial velocity.

Fig. 5 shows pressure along the channel. In condition of A1 analyzed, pressure gradient was smaller than Q1D design that caused decelerating trend of flow over the channel. Meanwhile in condition of A2 analyzed, trend of pressure gradient was similar to Q1D design, but velocity was lower than Q1D design because of different geometry. Such like trend turned down output.

Fig. 6 shows Faraday current. Faraday current of 3D-analysis was lower than Q1D design, because velocity and geometry were different from Q1D design.
Fig. 7 shows conductivity along the channel. Compare result of 3D-analysis with Q1D design, conductivity of 3D-analysis was higher except near the inlet. It said that temperature along the channel higher than Q1D design.

In the next place, distribution of some parameters at cross section is shown only A2 condition.

Fig. 8 shows hall electric field. Compare result of 3D-analysis with Q1D design, hall electric field of high along the channel. This is clear from Ohm’s low.

Figs. 8(a), (b) and (c) shows distribution of temperature at cross plane 3m, 6m and 9m from inlet, respectively. (a) and (b) shows smooth distribution near both electrode. But (c) shows a obviously different distribution on either end of cathode. (c) shows small two peaks either end of cathode because of secondary flow shown Fig. 9.

**Fig. 8 Distribution of temperature at y-z plane**

**Fig9 secondary flow at y-z plane**

**Fig10 Distribution of hall current at y-z plane**

3D-analysis was smaller, because conductivity was...
cross plane 3m, 6m and 9m from inlet, respectively. Figs. 9 shows the development of secondary flow. From this Figs. secondary flow evolves from 6m to 9m. At 9m, completely secondary flow rises on either end of cathode. According to this phenomenon, distribution of temperature has tow peaks near the cathode.

Figs. 10 (a), (b) and (c) shows hall current at cross plane 3m, 6m and 9m from inlet, respectively. It turns out that hall electric current is negative at channel center and positive near the electrode from Figs. 10. Also figs. 10 shows development of hall electric current, hall electric current has small peaks near the cathode as near outlet. This small peaks driven secondary flow.

**Conclusion**

In this study, it turned out that three-dimensional phenomena raised in 500MWth faraday MHD channel with coal synthesized and preheated gas combustion. The fact caused that the gas has strong interaction. And, electrode wall temperature was set 2000K so that conductivity in the boundary layers was kept high. This point strengthened MHD interaction additionally. Especially pertaining to hall electric field, hall electric field is small in spite of large faraday current along the channel. From this point, this MHD generator has low possibility to raise dielectric breakdown.

But in this study, 3D-analysis is only preliminary analysis. Control volume of 3D analysis was too large to analysis phenomena of MHD interactions, so it is need to analyze in detail.

**Nomenclature**

- $R$ = gas constant
- $A$ = cross section area
- $m_0$ = mass flow
- $f$ = friction loss
- $q$ = heat loss
- $\sigma$ = conductivity
- $\delta$ = thickness of boundary layer
- $G$ = G factor
- $C_1, C_2, C_D$ = constant in the turbulence model
- $E$ = electric field
- $G$ = generation of turbulence energy
- $h$ = enthalpy
- $J$ = current vector with components $J_x, J_y, J_z$
- $p$ = pressure field in cross-stream momentum equations
- $p$ = pressure field in axial momentum equation
- $q_x, q_y, q_z$ = heat fluxes in the $x, y, z$ direction
- $u, v, w$ = velocities in the $x, y, z$ direction respectively
- $U$ = velocity vector
- $x, y, z$ = coordinate direction
- $\beta$ = hall parameter
- $\varepsilon$ = dissipation of turbulence energy
- $\mu_l$ = laminar viscosity
- $\mu_r$ = turbulent viscosity
- $\rho$ = fluid density
- $\sigma_k, \sigma_{\varepsilon}, \sigma_h$ = turbulent Prandtl/Schmidt numbers for $k, \varepsilon, h$
- $\sigma_i$ = laminar Prandtl number for enthalpy
- $\tau_{ij}$ = shear stress in the $i$ direction, on the plane perpendicular to $j$ direction
- $\phi, \phi^*$ = electrical potentials

**References**

1. Naoyuki Kayukawa “Gas Turbine and Open-Cycle MHD Generator for CO2-free Power Generation System” AIAA-2002-2209