Numerical Simulation on Non-equilibrium Plasma Flow in an Annular Hall MHD Device

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Abstract

The characteristics of the annular Hall MHD device under the wide range of the operating conditions and physical phenomena in the device with the electrical energy extraction and addition are investigated in detail by carrying out time-dependent three-dimensional numerical simulations. This device has a cylindrical shape and has such an interesting feature from the engineering point of view as that this device can work both as the generator and the accelerator. The results show that under the high power (loading) operation, the discharge becomes steady, however, under the low power (loading) operation, the lack of Joule heating induces the unsteady discharge. This unsteady discharge has a clockwise spiral structure, and rotates clockwise at the frequency of ~ 30[kHz]. Under the condition of adding electrical energy into the plasma, it was found that effective acceleration cannot be obtained if two modes of the plasma ('generation plasma' and 'acceleration plasma') co-exists in the device.

Keywords non-equilibrium MHD generation, MHD accelerator, annular Hall MHD device, energy extraction, energy input, energy conversion

Introduction

The magnetohydrodynamic energy conversion can be categorized as (1) extraction of the high energy of the working gas as the electrical energy, (2) conversion of the electrical energy into the kinetic energy of the gas by means of the external input of the electric power. The engineering application in the first case is expected as the generator (MHD power generation), and in the second case as the accelerator or electromagnetic propulsion.

The researches on the non-equilibrium MHD generator have been conducted at Tokyo institute of technology and its performance and the physical phenomena in the generator has been made clear both experimentally and numerically. The shape of the generator applied here is the 'disk-shape', sandwiching two disk walls. This 'disk-shape' is of great advantage to the generator because of the easiness of realizing the strong enough magnetic field perpendicular to the flow, however, is not applicable as the accelerator. On the other hand, as shown in Fig.1, the device which has a cylindrical shape (hereafter this device is referred to as 'annular Hall MHD device') has been proposed. Although this device has such an interesting feature from the engineering point of view as that this device can work both as the generator and the accelerator, the detail investigation on this device has not been made due to the difficulty of realizing the strong radial magnetic field needed for the efficient energy conversion.

Recently, the research and development on the MHD energy bypassed SCRAM jet engine has been carried out and the interests in the MHD acceleration is now increasing. Addition to that, this annular Hall MHD device has a possibility for the electro-magnetic propulsion. Therefore, in this study, carrying out the time-dependent three dimensional numerical simulations, the characteristics of the annular Hall MHD device under the wide range of the operating conditions and the physical phenomena in the device with the electrical energy extraction and addition are investigated in detail.

Basic Equations and Numerical Procedures

Basic Equations

Governing Equations for charged particles

A non-equilibrium MHD plasma consists of noble gas atoms, noble gas ions, seed atoms, seed ions and electrons. The governing equations for charged particles as follows.

(1) Ion Continuity Equation

$$\frac{\partial n_i^+}{\partial t} + \nabla \cdot (n_i^+ \vec{u}) = \dot{n}_i^+ = k_f n_e n_i - k_r n_e^2 n_i^+$$

$$n_e = \sum_i n_i^+, \quad (i = \text{Seed, NobleGas})$$
(2) Generalized Ohm’s Law

\[
\mathbf{j} + \frac{\beta}{|B|} \mathbf{j} \times \mathbf{B} = \sigma \left( \mathbf{E} + \mathbf{u} \times \mathbf{B} \right)
\]

(3) Electron Energy Equation

\[
\frac{j_i^2 + j_B^2}{\sigma} = 3n_e k(T_e - T_p) \sum_h \frac{m_e}{m_h} \bar{v}_{eh} \left( \frac{3}{2} kT_e + \epsilon_i \right) + \sum_i n_i^+ \left( \frac{3}{2} kT_e + \epsilon_i \right)
\]

\( h = \text{Seed, Noble Gas, Ion} \)

\( i = \text{Seed, Noble Gas} \)

As for the ionization rate term \( n_i^+ \) in eq.(1), ionization by the electron collisions and three body recombination are considered.\(^3\)\(^4\) The quasi-steady electron energy equation (3) described with two temperature model\(^5\) is adopted, since the relaxation time in electron temperature is shorter than that in electron density. In eq.(3), the electron radiation loss is neglected in this simulation because the electron radiation loss is much smaller than the collision loss by the order of \( 10^2 \sim 10^3 \).

**Governing Equations for Heavy Particles**

The time-dependent compressible Navier-Stokes equations with Lorentz force and Joule heating terms are given as follows.

(1) Continuity Equation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

(2) Momentum Equation

\[
\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{j} \times \mathbf{B}
\]

(3) Energy Equation

\[
\frac{\partial E}{\partial t} + \nabla \cdot (E \mathbf{u}) = \nabla \cdot \left( \mathbf{Q} \right) + \mathbf{j} \times \mathbf{B}
\]

\[
E = \rho \left( c_v T + \frac{1}{2} |\mathbf{u}|^2 \right)
\]

The coefficients of turbulent viscosity used in \( \tau_{ij} \) and thermal conductivity are calculated on the basis of Baldwin-Lomax method.\(^7\)

**Table 1 Operating conditions of annular Hall MHD device.**

<table>
<thead>
<tr>
<th></th>
<th>He - Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working gas</td>
<td>4.8 [MW]</td>
</tr>
<tr>
<td>Seed fraction</td>
<td>6.0 x 10^-5</td>
</tr>
<tr>
<td>Load resistance</td>
<td>0.1 ~ 4.0 [Ω]</td>
</tr>
<tr>
<td>Applied voltage</td>
<td>0.1 ~ 1.5 [kV]</td>
</tr>
<tr>
<td>Inlet swirl ratio</td>
<td>0.0</td>
</tr>
<tr>
<td>Magnetic flux density</td>
<td>0.75 ~ 3.0 [T]</td>
</tr>
<tr>
<td>Inlet stagnation temperature</td>
<td>2000 [K]</td>
</tr>
<tr>
<td>Inlet stagnation pressure</td>
<td>2.0 [atm]</td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>0.46 [kg/s]</td>
</tr>
</tbody>
</table>

**Maxwell Equations**

The conventional MHD approximations of charge neutrality and low magnetic Reynolds number are assumed. Then, Maxwell equations are reduced to

\[
\nabla \times \mathbf{E} = \mathbf{0}
\]

and

\[
\nabla \cdot \mathbf{B} = 0
\]

**Numerical Procedures**

The ion continuity equation (1), the continuity equation (4), the momentum equation (5), and the energy equation (6) of the hyperbolic type, which are transformed to a boundary fitted coordinate, are solved by the CIP method\(^6\) applied to curvilinear coordinate extended to the three-dimensional case. The non-linear electron energy equation (6) is solved by bisection method. The substitution of equation (2) into equation (8) reduces to the following elliptic equation:

\[
\frac{\partial}{\partial r} \left( \sigma r \frac{\partial \phi}{\partial r} \right) + \frac{\partial}{\partial \theta \left( \frac{\sigma}{1 + \beta^2} \left( 1 + \frac{\partial \phi}{\partial \theta} - \beta \frac{\partial \phi}{\partial z} - \beta u_B + u_z B \right) \right) + \frac{\partial}{\partial z \left( \frac{\sigma r}{1 + \beta^2} \left( \frac{\partial \phi}{\partial \theta} + \frac{\partial \phi}{\partial z} + u_B - \beta u_z B \right) \right) = 0
\]

using a potential function \( \phi \) defined from equation(7) as

\[
E_r = -\frac{\partial \phi}{\partial r}, \quad E_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}, \quad E_z = -\frac{\partial \phi}{\partial z}
\]

The elliptic equation is discretized with a finite difference method, and is solved by Bi-CGSTAB method.\(^6\) The electron temperature in the generator are obtained by solving the non-linear algebraic equation of (3) with bi-section method.

**Calculation conditions and boundary conditions**

The cross sectional view of the annular Hall MHD device and the coordinate system is shown in Fig.1. The magnetic field is applied in the \( r \)-direction spoke-like and a pair of anode and cathode is installed.
Fig. 1 Cross sectional view of the annular Hall MHD device and the coordinate system.

![Diagram](image)

Fig. 2 The distribution of magnetic flux density.

axisymmetrically upstream and downstream, respectively. The magnetic flux density has a distribution in r-direction as shown in Fig. 2. The distribution is simply given as an inverse proportional to radius, and the optimization of the magnetic field is not made in this simulation. The highest magnetic flux density is 3.0[T] at the radius of 20.0[mm] and the lowest is 0.75[T] at the radius of 80.0[mm].

The calculation region is covered from the inlet of the nozzle to exit of the channel along the z-direction, from 0 to π along the azimuthal(θ-) direction and between the walls in the normal(r-) direction. In the nozzle region (z = 0.0[mm]~34.0[mm]), Lorentz force is assumed not to be effective and only fluid properties are solved. The time step Δt is determined to be 0.04[μsec] and the number of grid points in r, θ and z directions are given as 31, 30 and 101 in the present simulation.

The boundaries of θ = 0 and θ = π are joined with the periodical condition. As for the inlet boundary condition for eq.(1), the ion number densities are given by assuming Saha equilibrium for the fixed inlet electron temperature of 3000[K]. All the independent gas properties at the inlet boundary of calculation region are fixed to the operating parameters listed on Table 1. At the exit boundary, all the physical properties are given by the free boundary condition, that is, differential values along the z-direction are zero.

Results and Discussion

The characteristics of annular Hall MHD device were investigated numerically by changing the external load resistance or external applied voltage under the operating conditions listed on Table 1. The voltage-current characteristics are shown in Fig. 3. In this figure, the positive voltage implies that the electrical energy is extracted from the plasma, thus, under this condition, this device works as the MHD generator. On the contrary, by applying the inverse voltage (the negative voltage), electrical energy can be input into the plasma and this device works as the MHD accelerator. In the following sections, the operating characteristics of the device and the physical phenomena under the energy extraction and the energy input are discussed.

Operating characteristics of annular Hall MHD device under the energy extraction

The enthalpy extraction ratio (=electrical output power/thermal input) and isentropic efficiency are plotted against the load resistance in Fig. 4. As seen in Fig. 4, the highest enthalpy extraction ratio of 4.5[%] and the highest isentropic efficiency of 14.5[%] are both obtained for the load resistance of 2.0[Ω]. These efficiencies are not so high for the MHD electrical power generation. This is because the optimization of the channel shape and the operating conditions are not perfectly made. It can be feasible to improve the performance by doing so.

Figure 5 shows the r-z two-dimensional distributions of electron temperature. From this figure, it is found that the electron temperature has the nonuni-
Fig. 4  Enthalpy extraction ratio (E.E.) and isentropic efficiency (I.E.) as a function of external load resistance.

Fig. 5  $r - z$ two-dimensional distribution of electron temperature under the condition of the external load resistance of $2.0[\Omega]$.

form distribution in $r$-direction. This is caused by the decreasing magnetic flux density in $r$-direction and is explained as follows. The Hall parameter becomes smaller near the outer wall as the magnetic flux density decreases along the radius, while the electron conductivity has almost an uniform distribution in $r$-direction. Therefore, the local internal resistance of the plasma becomes smaller along the radius, in other words, the local loading parameter becomes higher along the radius. As a result, large Faraday current density is induced and larger Joule heating can be obtained near the outer wall, hence, the electron temperature increases along the $r$-direction. Addition to that, under this condition, the discharge structure has an uniformity in $\theta$-direction.

Figure 6 shows the axial profile of main flow velocity ($at r=50[mm]$). The $r - z$ two-dimensional distributions of normalized axial flow velocity under the non-MHD effect and under the load resistance of $2.0[\Omega]$ are shown in Fig.7 and 8, respectively. Note that the axial flow velocity is normalized with the main flow velocity at each position $z$. The fluid-dynamical structures such as the large separation of the boundary layer or pseudo-shock wave are not observed, and the main flow velocity keeps almost constant value in the device as found in Fig.6. These results suggests that the interaction between the plasma and the fluid was very weak under this condition and stronger interaction is desirable for the effective extraction of the energy from the working gas. From Fig.8, comparing the case with no MHD effect(Fig.7), it is seen that the boundary layer both on the inner and outer wall develops remarkably even under this weak MHD interaction. The strong magnetic flux density near the inner wall causes strong Lorentz force against the working gas. The working
gas is also decelerated by the strong Lorentz force attributed to the large Faraday current near the outer wall. A large inverse pressure gradient is induced by the strong Lorentz force near the walls, which results in the further development of the boundary layer.

It is found from Figs. 3 and 4 that the output power decreases for the load resistances lower than 2.0[Ω]. For the lower load resistances (≤ 0.5[Ω]), large enough Joule heating to lead to the fully ionized seed plasma cannot be obtained at the inlet of the device. This lack of Joule heating induces the spatially nonuniform distribution of electron number density in θ-direction at the inlet, which is caused by the ionization instability. Under this condition, the discharge becomes unsteady, and forms the nonuniform structure in θ-direction as shown in Fig. 9. This unsteady discharge has a clockwise spiral structure, and it rotates clockwise at the frequency of ~ 30[kHz].

Operating characteristics of annular Hall MHD device under the energy addition

As found in Fig. 3, in the case the applied voltage less than ~0.1[kV], sufficient Joule heating cannot be obtained, thus the discharge becomes unsteady as mentioned in the previous section. While, by applying higher voltage, the electron temperature in the device rises and the steady discharge is maintained.

Figures 10 and 11 show the profiles of the main axial flow velocity and the potential in the device in z-direction under the applied voltages of 0.15[kV] and 1.5[kV], respectively. In the case of the applied voltage of 0.15[kV], the gradient of the potential becomes positive (negative electrical field) in the upstream region.
Fig. 12 $r-z$ two-dimensional distribution of electron temperature under the condition of applied voltage of 1.5[kV].

Fig. 14 $r-z$ two-dimensional distribution of heavy particle temperature under the condition of applied voltage of 1.5[kV].

Fig. 13 $r-z$ two-dimensional distribution of normalized axial flow velocity under the condition of applied voltage of 1.5[kV].

Fig. 15 Lorentz efficiency as a function of externally applied voltage.

and it becomes negative (positive electrical field) in the downstream region. The former corresponds to the 'power generation plasma' and the latter to the 'non-generation (acceleration) plasma'. If the two modes of the plasma co-exists in the device as seen in Fig.11, the working gas is decelerated by the Lorentz force attributed to the negative electric field in the upstream region, so that, under this applied voltage, the no more the velocity attained by the fluid-dynamical expansion is obtained. On the other hand, under the applied voltage of 1.5[kV], the gradient of the potential becomes negative throughout the channel, the working gas is accelerated by the externally applied electrical energy.

Figure 12 shows the $r-z$ two-dimensional distributions of electron temperature under the condition of applied voltage of 1.5[kV]. The Electron temperature in the device locally rises up to 24000[K] because of the strong electric field and the high velocity of the gas. The electron temperature becomes locally higher in downstream near the outer wall as seen in this figure. Since the working gas is accelerated and has a relatively high velocity in the downstream, the Joule heating attributed to the electro-motive force becomes large. This is the reason why the locally higher electron temperature is generated in the downstream. The phenomenon that the electron temperature becomes higher near the outer wall can be explained in the same way as found in the previous section.

The $r-z$ two-dimensional distributions of normalized axial flow velocity and heavy particles temperature are shown in Figs.13 and 14, respectively. The translational energy of the electrons is transferred to the heavy particles through collisions. As a result, the temperature of heavy particles becomes very high, es-
especially in the downstream near the outer wall. This heat addition to the fluid causes the further development of the boundary layer, because the effect of viscosity becomes large in the high temperature region. This mechanism of the development of the boundary layer is different from that under the energy extraction. Moreover, the gas near the outer wall is accelerated by the strong Lorentz force attributed to the large Faraday current density. The strong Lorentz force also pushes the gas near the inner wall owing to the strong magnetic field, therefore, the gas near the walls is accelerated and the separation of boundary layer can be suppressed in this configuration of the device.

Figure 15 shows the Lorentz efficiency as a function of the applied voltages. The Lorentz efficiency is defined as:

\[
\eta_L \equiv \frac{\text{Work done by the Lorentz force}}{\text{Total input power}} = \frac{\iiint \mathbf{u} \cdot (\nabla \times \mathbf{B}) \, dV}{V I} (11)
\]

\[
= 1 - \frac{\iiint |\mathbf{j}|^2 \, dV}{\sigma V I} (12)
\]

It is understandable from this figure that an optimum applied voltage exists. This existence of the optimum applied voltage can be explained as follows. For the low applied voltages, sufficient electrical conductivity can not be obtained owing to the lack of Joule heating. Thus, Faraday current providing acceleration force is not so high, which results in the low Lorentz efficiency. For the high applied voltages, most of the input power is transferred not to the kinetic energy of the working gas, but to the heavy particles' internal energy through an excessive Joule heating. This fact also leads to the reduction of Lorentz efficiency.

**Conclusions**

The characteristics of the annular Hall MHD device under the wide range of the operating conditions and physical phenomena in the device with the electrical energy extraction and addition are investigated in detail by carrying out time-dependent three-dimensional numerical simulations. The researches aiming the improvement of the device and the optimization of the operating conditions are left as the future work.

**References**


