This paper presents simplified analyses of MHD thrust augmentation for rocket and airbreathing primary propulsion systems. The basic idea behind MHD thrust augmentation is the redistribution of available on-board energy. Typically, this is the chemical energy of the fuel or propellants. In principle, total propulsive performance may be enhanced by extracting enthalpy (energy) from the flow in one stage of the engine cycle and transferring it to a different stage of the engine cycle. This paper examines two unique jet propulsion systems, one rocket based and one airbreathing based, which utilize MHD as an energy transfer mechanism for thrust augmentation.

First, the performance potential of the a Rocket-Induced MHD Ejector (RIME) engine concept for redistribution of momentum and energy between a primary rocket and a secondary by-passing air flow was organized using a simplified first order thermodynamic analysis. The flow processes were organized in full analogy with rather well known systems in conventional gasdynamic applications.

Second, it has been proposed (AJAX Concept) that MHD generator/accelerator coupling can significantly improve the performance of airbreathing ramjet/scramjet engines during operation at hypervelocity flight speeds. Here again, the basic idea is to transfer energy from one stage of engine cycle to another in order to minimize the entropy rise of the total aircraft system. This energy management objective is accomplished by extracting a portion of the aerodynamic heating energy at the engine inlet and converting it to kinetic energy in the engine exhaust. In this study, our goal was to assess the physical and thermodynamic feasibility of an AJAX type propulsion system for hypersonic flight.

Introduction

In this report a simplified analysis of the thrust augmentation of jet propulsion systems both for rocket and airbreathing type of primary thrusters is presented. The general idea of such an augmentation technique is so called a momentum transformation during the motion through fluid environment (atmospheric air, ocean water or others similar).

A more particular case of an application of such an idea to jet propulsion of launching operation system has been proposed and preliminary discussed by J.Cole et.al in Ref.1. In this paper an MHD generator/accelerator coupling was proposed as a tool providing the momentum and energy redistribution between a primary rocket engine and a secondary by-passing flow. The latter assumed is organized in full analogy with the rather well known systems in conventional gasdynamics applications.

The utilization of MHD generator/accelerator coupling for a jet propulsion thrust manipulation was considered and earlier, for example, to improve significantly performance of ramjet/scramjet operation in supersonic flight speeds. Probably, the first of such a proposal known as AJAX concept has been proposed and developed in Russia (see Ref.2). Unfortunately a few studies of both mentioned above and any others similar to those is available now in literature.

The primary goal at this of this study is to assess physical and thermodynamics background for such an application.

Simplified theory of a by-pass augmentation of a rocket engine thrust

Let us consider the thrust of a system presented in Fig.1 taken from Ref.1. In this Chapter any non-gasdynamics components given in this picture are neglected. Thus a conventional gasdynamics by-pass thrust augmentation system is considered.

In this case the rocket jet is a ejecting gas. The ejected gas is the ambient air coming into mixing chamber directly from atmosphere, and the mixed flow is ejected into atmosphere again. Let us assume for simplicity that the nozzle and diffuser are designed in such a way that the static pressure at the nozzle exit and at the exhaust of the mixture from the diffuser are equal to the ambient pressure $p_a$. In this case the effective thrust acting on the whole ejector system being in the rest in respect to ambient media is

$$T_p = (m_p + m_a)V_{ex}$$

where $m_p$ and $m_a$ are the mass flow rates of the rocket propellant and the ejected air,
correspondingly, and \( V_{\text{ex}} \) is the mixture velocity at the diffuser exit.

For comparison note that the thrust of the rocket engine alone at the assumed thermodynamics parameters in the combustion chamber is

\[
T_p = m_p V_p,
\]

where \( V_p \) is the rocket ejection velocity.

Due to so called the impact losses the kinetic energy of the mixture is less the initial kinetic energy of the propellant flow

\[
\frac{m_p + m_a V_{\text{ex}}^2}{m_p} V_p^2 = \eta < 1.
\]

The value \( \eta \) can be treated as an ejector efficiency. From the previous expression one can easily obtain that

\[
\frac{m_p + m_a V_{\text{ex}}^2}{m_p} = \sqrt{\frac{m_p + m_a}{m_p}} V_p \Rightarrow \eta = \frac{m_p + m_a}{m_p}
\]

or

\[
\frac{T_{\text{eff}}}{T_p} = \sqrt{1 + \frac{f}{\eta}},
\]

where \( f \) is air-to-propellant ratio.

The latter expression means that for given value of the ejector efficiency \( \eta \) the sufficient amount of air can be added to the propellant flow so that \( 1 + f > 1/\eta \) and, consequently, \( T_{\text{eff}} / T_p > 1 \).

Basing on a standard approach of the conservation laws applied to the properly chosen control volume one can derive the final expression of the thrust augmentation of the rocket engine

\[
\eta = \frac{T_{\text{eff}}}{T_p} = \frac{1}{\sqrt{1 - \alpha^2}} \left[ 1 - \frac{\alpha}{\varphi + 1} \left( f + 1 \right)^2 - f \alpha \right],
\]

where

\[
\alpha = \frac{A_{\text{rocket}}}{A_{\text{by-pass}}}, \quad \varphi = \frac{A_{\text{exit}}}{(A_{\text{rocket}} + A_{\text{by-pass}})}, \quad \omega = \frac{V_{\text{flight}}}{V_p}.
\]

and for air-to-propellant ratio the following expression can be obtained (for details see for example Ref.3)

\[
f = \frac{\varphi (1 + 1/\alpha) \sqrt{2 \alpha + \alpha^2 \varphi^2 (\varphi^2 - 1) \left( 1 + \alpha^2 \varphi^2 \right) \omega^2} - 1 - \varphi^2}{1 + \alpha^2 \varphi^2}
\]

It can be shown that the thrust augmentation coefficient \( \eta \) and by-pass coefficient \( f \) are monotonic functions on geometry parameters \( \alpha \) and \( \varphi \). It is important that \( \eta > 1 \) and can reach a rather high value. Thus, for the system resting in respect to the ambient atmosphere the thrust augmentation can be very significant. The maximum of the thrust augmentation coefficient \( \eta = 2\varphi \) is defined by the exit-to-entrance ratio and for \( \varphi = 1 \) can be as high as 2.

When the propulsion system is moving in respect to the ambient atmosphere the thrust augmentation coefficient is always less. When the flight velocity approaches sonic and supersonic values the shock wave losses result in fast reduction any positive effect.

Thus, the expectation of the thrust augmentation of a rocket engine with increasing value of the by-passing coefficient is probably overestimated by neglecting of significant mechanisms of losses.

2. General concept of MHD assisted rocket thrust augmentation

2.1. Reference case

The conventional case of the thrust production system is a jet propulsion system converting the chemical energy of a fuel+oxidant mixture into the heat of the (compressed) gaseous combustion products. The expansion of this combustion product from the initial (combustion chamber) pressure to the pressure approximately equal to the ambient pressure (the best case then both are equal to each other) through the nozzle of a proper shape results in conversion of the heat energy into the kinetic energy of the jet. It can be easily shown that such an energy transformation results in the thrust acting on the jet creating system. When the whole amount of fuel and oxidant is located on-board of the system the case of the rocket jet propulsion is presented. When the ambient air is used as an oxidant to support of combustion process releasing the chemical energy of the fuel the case of airbreathing propulsion is presented.

Two main type of jet propulsion will be considered below: a rocket engine and ramjet/scramjet as an example of airbreathing device.

A general expression of the thrust valid for both cases is

\[
f = \frac{\varphi (1 + 1/\alpha) \sqrt{2 \alpha + \alpha^2 \varphi^2 (\varphi^2 - 1) \left( 1 + \alpha^2 \varphi^2 \right) \omega^2} - 1 - \varphi^2}{1 + \alpha^2 \varphi^2}
\]
Here  and  are mass flow rate of air and propellant component moving with propulsion system,  and  undisturbed air flow velocity in respect to the propulsion system under consideration and (mass averaged) jet velocity at the exit of the propulsion system. When the only driving force to accelerate the propellant through the nozzle is heat energy of combustion products the maximal achievable exhaust velocity can be defined as

\[
V_{ex,max} = \left( \frac{2C_p T_{stag,p}}{C_p T_{ex} + q_{fuel}} \right)^{\gamma}.
\]

(As always in this simplified analysis the ideal gas properties are assumed for all cases.) The air-to-fuel ratio \( f = \frac{m_a}{m_p} \) is an important parameter in this analysis.

In particular case of a rocket engine \( m_a = 0 \) and \( f = 0 \).

The thrust is maximal when the expansion process is completed at the pressure equal to ambient pressure \( p_a \). For the ideal (isentropic) expansion the temperature drop is defined by

\[
\Delta T = \frac{T_{stag.extr} - T_{stag,p}}{T_{stag,p}} = \left( \frac{1}{\gamma} \right)^{\frac{1}{\gamma-1}} \left( \frac{p_{stag.extr}}{p_{stag.p}} \right)^{\frac{\gamma}{\gamma-1}} - 1.
\]

The rocket jet thrust loss due to the enthalpy extraction is as following

\[
\delta T_p = -T_p \left[ 1 - \left( \frac{1 - \eta_{extr}}{1 - \epsilon_p} \right) \left( \frac{1 - \epsilon_p \eta_{extr}}{\eta_i - \eta_{extr}} \right) \right].
\]

For the further analysis it is important to define the behavior of these parameters when the enthalpy extraction ratio goes to zero. One can find for this case
\[ V_{\text{ex},\text{res}} = V_p \sqrt{\left(1 - \eta_{\text{extr}}\right)\left(1 - \frac{\varepsilon_p \eta_{\text{extr}}}{1 - \varepsilon_p}\right)} \]
\[ = V_p \left[ 1 - \frac{\eta_{\text{extr}}}{2} \left(1 + \frac{\varepsilon_p}{1 - \varepsilon_p} \frac{1}{\eta_{\text{stg},\text{a}}} \right) \right] . \]

### 2.3. Thrust benefit

The total enthalpy extracted from the rocket jet is supplied afterwards into the flow of ambient air (surrounding fluid). Let us define the device providing such an energy transfer into the air flow as an \textit{accelerator}. This device transferring the energy to the air flow can produce some mechanical work (acceleration or/and compression) and heating.

\[ T_{\text{stg},\text{a},\text{sup}} = T_{\text{stg},\text{a}} + \eta_{\text{extr}} \frac{Q_{\text{prop}}}{m_p c_p} = T_{\text{stg},\text{a}} (1 + \Lambda) , \]

with

\[ \Lambda = \frac{\eta_{\text{extr}} Q_{\text{prop}}}{m_p c_p T_{\text{stg},\text{a}}} . \]

The final result of a thrust augmentation depends strongly upon the ratio of these two contributions. It is convenient again to characterize such an energy conversion in terms of the process efficiency defined in full analogy with the preceding analysis of the thrust losses as a ratio of the enthalpy transferred in an ideal (isentropic) process with the stagnation pressure change given to the those in real process within the same stagnation pressure change interval, thus

\[ \eta_Q = \frac{\Delta H_{Q,\text{isentropic}}}{\Delta H_{Q,\text{actual}}} \]

and

\[ p_{\text{stg},Q} = p_a \left( \frac{T_{\text{stg},\text{a}}}{T_a} \right)^{\gamma} \left(1 + \Lambda \eta_{\text{extr}} \eta_Q\right)^{\gamma} = p_{\text{stg},\text{a}} \left(1 + \Lambda \eta_{\text{extr}} \eta_Q\right)^{\gamma} \]

It should be noted that for given definition of \( \eta_Q \) the latter can varies from \( \eta_Q = 1 \), corresponding to ideal isentropic compression till a negative value corresponding to a strong dissipative process resulting in \( p_{\text{stg},Q} < p_a \).

The expansion factor is now equal to

\[ \varepsilon_Q = \left( \frac{p_a}{p_{\text{stg},Q}} \right)^{\gamma/(\gamma-1)} = \frac{1}{B_a \left(1 + \Lambda \eta_{\text{extr}} \eta_Q\right)} \]

Applying a conventional procedure of the exhaust velocity definition one can obtain consequently

\[ V_{a,Q} = \frac{V_{\text{max},Q}}{1 - \varepsilon_Q} = V_a \sqrt{\left(1 + \Lambda \frac{\varepsilon_a}{1 - \varepsilon_a} \eta_{\text{extr}} \eta_Q\right)} , \]

An important case \( \Lambda \to 0 \) (low enthalpy extraction) the preceding expression can be simplified as

\[ V_{a,Q} \big|_{\Lambda \to 0} \to V_a \left[ 1 + \frac{\Lambda}{2} \left(1 + \frac{\varepsilon_a}{1 - \varepsilon_a} \eta_{\text{extr}} \eta_Q\right) \right] , \]

that leads to

\[ T_{a,Q} = m_a V_{a,Q} - m_p V_a \to \]

\[ \to m_a V_a \Lambda \left(1 + \frac{\varepsilon_a}{1 - \varepsilon_a} \eta_{\text{extr}} \eta_Q\right) \approx \]

\[ \approx m_p V_p \left(1 - \frac{\varepsilon_a}{1 - \varepsilon_a} \eta_{\text{extr}} \eta_Q\right) \]

with additional assumption that the compression factor for the primary rocket jet propulsion system small enough, i.e.

\[ \varepsilon_p << 1 \].

### 2.4. Total thrust

The total thrust augmentation is now defined as a sum of the residual primary rocket engine thrust and benefit with extra thrust provided by by-passing accelerated air flow.

\[ T_z = T_{a,Q} + T_{\text{res}} . \]

The general expression is too complicated for a fast analysis. For this reason let us consider the case of low enthalpy redistribution bearing in mind that this case provides the most optimistic results.

\[ T_z = T_{a,Q} + T_{\text{res}} \big|_{\eta_{\text{ex}} \to 0} \to \]

\[ \to m_p V_p \left(1 - \frac{\varepsilon_a}{1 - \varepsilon_a} \eta_{\text{extr}} \eta_Q\right) \approx \]

\[ \approx m_p V_p \left(1 - \frac{\varepsilon_a}{1 - \varepsilon_a} \eta_{\text{extr}} \eta_Q\right) \]

with the additional assumption of the previous section.

Here \( \omega = V_p / V_a \) is a velocity ratio. It is clear that the positive augmentation effect can expected only for cases of relatively low the flight speed.

Of course, more reliable conclusion needs accurate calculation.
2.5. T-S diagram cycle analysis

The thermodynamics cycle characteristics can be qualitatively explained with simplified T-S diagram technique. The ideal gas properties are assumed as always.

In Fig.2 the T-S diagram cycle analysis from Ref.6 is presented. The cycle (dashed lines) represents a reference cycle of a rocket engine driven by a chemical propellant (see Fig.3). The heat supply to the propellant in combustion process assumed as isobaric process with pressure $p_1=p_{comb}$. The expansion process in the nozzle from the combustion pressure $p_{comb}$ to the pressure assumed for simplicity equal to ambient pressure $p_B$. The curve closing a cycle represents the ejected gas cooling process in the atmosphere. The nozzle efficiency is typically rather high so the expansion curve is very close to vertical (isentropic expansion) line. In this case the thrust (per 1kg/1sec mass flow rate of propellant) is defined by the exhaust velocity $V_{ex}$. The velocity square i.e. equal to doubled kinetic energy of the ejected flow which equal to enthalpy difference at points upper and loner points respectively.

The process in questions involves two different working media: primary propellant and ambient air. For this reason the following ‘double media’ analysis is even more qualitative.

Let us consider the primary propellant cycle. It starts again from the same initial point and goes along the same heating curve. Further the curve from $p_1=p_{comb}$ to $p=p_3$ represents the process of enthalpy extraction from the primary propellant jet. It is qualitative outlined that such a process could be very dissipative so the entropy growth can be significant. The important consequence of such low efficiency is a significant stagnation pressure drop in comparison with ideal isentropic expansion. The energy extracted in this process is represented by the enthalpy drop between points upper and loner. The following nozzle expansion process from (from an intermediate pressure $p_3$ to the ambient pressure $p_B$) provides some residual thrust by primary propellant defined by the kinetic energy of the exhausting flow. This kinetic energy is equal to the enthalpy drop.

Note that the sum of energy extracted in the process of Enthelpy extraction and kinetic energy of the exhausting flow is less then the total enthalpy drop in the reference case due to lower efficiency assumed for energy extraction process.

The ambient air acceleration cycle is presented in the diagram by doubled solid curves. Assuming a rather high efficiency of air compression in the intake system one can define the compression process as a vertical line from $p_B$ to $p=p_2$. So the enthalpy at the upper point represents approximately the stagnation enthalpy of the flow. The process of injection of the energy earlier extracted from the primary propellant jet to the air flow is presented alternatively by two curves. The difference between these cycle trajectories is defined by isentropic efficiencies along each trajectory. So, the first case is an ideal isentropic process, the second represents a process can be called as ‘heating’ and less desirable process. The expansion process in a by-passing nozzle is represents by one of alternative lines between one the upper point and the corresponding point at ambient pressure isobar. The thrust defining value is now the difference of the incoming air velocity $V_a$ and the exhausting air velocity $V_{ex}$. Note that the kinetic energy difference available in such an expansion process is equal to the injected energy only in the case of the maximal cycle efficiency of the energy injection).

The mass flow rate correlation has to be included into this analysis also.
The conclusion is not obvious. The final result can be as positive and negative as well.

3. The simplified thermodynamics analysis of an ajax type concept

3.1. Input data and basic definitions

Let us assume that the working media is an ideal gas with constant specific heat capacities \( C_p \) and \( C_v \), so the specific heat capacity’s ratio \( \gamma = C_p/C_v \) is also constant.

Let us introduce several values to characterize the process efficiency following after standard approach given in textbooks (see, for example, Ref.4).

Compression (or diffusion) efficiency to characterize the efficiency of the compression process in the intake diffuser of a ramjet/scramjet engine

\[ \eta_D = \frac{\Delta h_D^{\text{isentropic}}}{\Delta h_D^{\text{actual}}} \]

and similar one for the expansion process through the nozzle

\[ \eta_{ex} = \frac{\Delta h_{ex}^{\text{isentropic}}}{\Delta h_{ex}^{\text{actual}}} \]

Energy conversion in the MHD generator/accelerator can be also characterized by thermodynamics efficiencies that define the ratio of the total enthalpy change along the process to the those under isentropic condition assumed, thus

\[ \eta_{MHDgen} = \frac{\Delta h_{MHDgen}^{\text{actual}}}{\Delta h_{MHDgen}^{\text{isentropic}}} \]

and

\[ \eta_{MHDacc} = \frac{\Delta h_{MHDacc}^{\text{actual}}}{\Delta h_{MHDacc}^{\text{isentropic}}} \]

Due to the fact that the entropy can growth through the combustion process occurring in the flowing gas mixture under the constant static pressure condition let us define also the combustion efficiency as a ratio of the stagnation pressure at the inlet of the combustion chamber to the stagnation pressure at the combustion chamber outlet

\[ \eta_{comb} = \frac{P_{\text{stag,flow}}}{P_{\text{stag,rest}}} \]

One of the main feature of ramjet/scramjet operation in supersonic flight is the rather high stagnation temperature at the combustion chamber inlet which is practically equal to stagnation temperature of the free stream

\[ T_{\text{stag,D}} = T_{\text{stag,a}} = T_a \left( 1 + \frac{\gamma - 1}{2} M_a^2 \right) \]

Some fuel heat is also added to the gas mixture the temperature in the combustor can reach the level unacceptable for a device operating in long duration (steady state) mode. Thus, the temperature limitation should be introduced into consideration as the condition, that the stagnation temperature of the flow at any location from the intake inlet till outlet nozzle must be not higher then the limiting temperature defined by a designer

\[ T_{\text{stag,flow}} \leq T_{\text{limit}} \]

Under these definitions and assumptions let us consider the process performances in natural consequence.

3.2. Compression

The ideal values of the free stream are the free stream stagnation pressure

\[ P_{\text{stag,a}} = P_a \left( 1 + \frac{\gamma - 1}{2} M_a^2 \right)^{\gamma/\gamma-1} \]

the free stream stagnation temperature

\[ T_{\text{stag,a}} = T_a \left( 1 + \frac{\gamma - 1}{2} M_a^2 \right) \]

the free stream Mach number

\[ M_a^2 = \frac{V_a^2}{\gamma R T_a} \]

with \( P_a \), \( T_a \), \( V_a = V_{\text{flight}} \) the static pressure, the static temperature and the free stream velocity. The latter is equal to the flight velocity.

Using the diffusion efficiency introduced earlier one can obtain the stagnation pressure at the end of the inlet diffuser as follows

\[ P_{\text{stag,D}} = P_{\text{stag,a}} \left( 1 + \frac{\gamma - 1}{2} M_a^2 \eta_D \right)^{\gamma/\gamma-1} \]

With no heat losses through the wall the stagnation temperature before the combustion chamber is equal in this case to the free stream stagnation temperature

\[ T_{\text{stag,D}} = T_{\text{stag,a}} \]

3.3. Combustion

In the combustor the stagnation temperature of the incoming flow is to increase due to a fuel heat release.

\[ h_{\text{stag,comb}} = h_{\text{stag,D}} + q_{\text{fuel}}/f = C_p T_{\text{stag,comb}} \leq C_p T_{\text{limit}} \]

In preceding expression \( q_{\text{fuel}} \) represents the specific heat of the fuel under stoichiometry condition and \( f \) is the air-to-fuel ratio. For given stagnation conditions and fuel characteristics...
there is the only possibility to satisfy to the right inequality it is to change air-to-fuel ratio \( f = \frac{q_{\text{fuel}}}{C_p T_{\text{limit}} - h_{\text{stag},D}} \).

Even more for certain flight conditions when \( h_{\text{stag},D} \geq C_p T_{\text{limit}} \)
no combustion (fuel injection) is possible because of the combustor temperature limitation. This means obviously that no positive thrust can be produced in such a jet propulsion system.

Meanwhile the limitation conditions is acceptable to produce some thrust let us continue the analysis of the cycle.

In practice the combustion process occurs in the flow with some Mach number greater than zero. It means that the static temperature in the combustion chamber is less then stagnation temperature that results in additional losses of the stagnation pressure. The estimation of these losses (made with approach given in Ref.5) is

\[
\frac{\delta p_{\text{stag}}}{p_{\text{stag}}} = \frac{\delta T_{\text{stag}}}{T_{\text{stag}}} = \frac{1}{2} M_{\text{stag}}^2 \frac{\delta T_{\text{stag}}}{T_{\text{stag}}} \leq \frac{1}{2} \frac{M_{\text{stag}}^2}{M_{\text{stag}}} \frac{T_{\text{limit}} - T_{\text{stag}}}{T_{\text{limit}}} .
\]

For RAMJET \( M_{\text{stag}} < 1 \) and for SCRAMJET \( M_{\text{stag}} > 1 \) operating modes that can create again some problem in realization such a process. Anyway, the efficiency \( \eta_{\text{comb}} \) should be positive that results in an inequality

\[
\frac{T_{\text{limit}} - T_{\text{stag}}}{T_{\text{limit}}} \leq \frac{1}{2} \frac{M_{\text{stag}}^2}{M_{\text{stag}}} .
\]

The latter looks like a very difficult condition for high speed flight operation.

Thus the combustion yields outlet flow parameters

\[
T_{\text{stag},\text{comb}} \leq T_{\text{limit}}, \quad P_{\text{stag,comb}} = P_{\text{stag,D}} \eta_{\text{comb}} = P_{\text{stag,D}} \left( 1 - \frac{1}{2} \frac{M_{\text{stag}}^2}{M_{\text{stag}}} \frac{T_{\text{limit}} - T_{\text{stag},D}}{T_{\text{limit}}} \right) .
\]

In previous expression the values \( P_{\text{stag,D}}^* \) and \( T_{\text{stag,D}}^* \) are reserved for the MHD assisted case (see section 5). With no MHD assist \( P_{\text{stag,D}} = P_{\text{stag,D}}^* \), consequently \( \eta_D^* = \eta_D \), and \( T_{\text{stag,D}}^* = T_{\text{stag,D}} \), so

\[
P_{\text{stag,comb}} = P_{\text{stag,D}} \left( 1 + \frac{1}{2} \frac{M_{\text{stag}}^2 \eta_D}{M_{\text{stag}}} \right) \frac{\gamma}{\gamma - 1} \left( 1 + \frac{1}{2} \frac{M_{\text{stag}}^2}{M_{\text{stag}}} \right) \frac{P_{\text{stag},\text{comb}}}{T_{\text{stag},\text{comb}}} \frac{T_{\text{limit}} - T_{\text{stag},D}}{T_{\text{limit}}} .
\]

It is convenient to define the expansion factor

\[
\varepsilon_{\text{comb}} = \left( \frac{P_{\text{stag},\text{comb}}}{P_{\text{stag,D}}} \right)^{\frac{1}{\gamma}} \left( 1 + \frac{1}{2} \frac{M_{\text{stag}}^2}{M_{\text{stag}}} \frac{\eta_D^{\gamma}}{\gamma - 1} \left( 1 + \frac{1}{2} \frac{M_{\text{stag}}^2}{M_{\text{stag}}} \right) \frac{T_{\text{limit}} - T_{\text{stag},D}}{T_{\text{limit}}} \right) \frac{T_{\text{stag},\text{comb}}}{T_{\text{limit}}} .
\]

3.4. Expansion

Using the expansion efficiency one can easily find that

\[
\Delta T_{\text{ex}} = \frac{V_{\text{ex}}^2}{2 C_p} = \eta_{\text{ex}} T_{\text{limit}} \left[ 1 - \varepsilon_{\text{comb}} \right] .
\]

The thrust reduced to the 1kg/sec mass flow rate of the propellant of such a propulsion system is equal to

\[
\frac{T}{m} = V_{\text{ex}} - V_{\text{a}} \frac{f}{f + 1} = \frac{2 T_{\text{limit}}}{C_p} \eta_{\text{ex}} \times \left( 1 - \frac{1}{f + 1} \right) \left( 1 + \frac{1}{2} \frac{M_{\text{stag}}^2 \eta_D^*}{M_{\text{stag}}} \right) \frac{T_{\text{limit}} - T_{\text{stag,D}}^*}{T_{\text{limit}}} \frac{V_{\text{a}}}{f + 1} .
\]

In this expression \( T_{\text{stag,a}} = T_{\text{stag,D}} \) and \( \eta_D^* = \eta_D \) if no MHD convertor is involved.
3.5. MHD assisted thrust augmentation of ram/scramjet

As it follows from results of the previous section too high stagnation temperature arisen due to high flight Mach number blocks any fuel injection and consequently no positive thrust can be produced. In order to solve this problem energy by-passing was proposed (Ref.2). The qualitatively the idea is rather simple: the first of all to extract some fraction of the total enthalpy of the incoming air flow before combustor, the second step is the combustion of the fuel in amount providing the limiting temperature at the combustor exit, the third step is to supply the enthalpy extracted in pre-combustor area back into the exhausting flow. The expected result is to increase the total thrust by extended fuel combustion when the flight conditions approaching the critical point \( \frac{T_{stag}}{T_{o}} \). Because of the high temperature environment the MHD energy conversion can be considered as a main candidate for such an energy by-passing mechanism. Disadvantages of MHD are rather low isentropic cycle efficiencies both for MHD generation and MHD acceleration.

Let us consider this process in more detailed.

Assuming that MHD generator can extract some power from the air flow. Then

\[
T_{stag,a}^{*} = T_{stag,a} - q_{MHDgen} \frac{C_p}{1 - \eta_{e,MHD}}
\]
or

\[
T_{stag,D}^{*} = T_{stag,D} \left( 1 - \eta_{e,MHD} \right).
\]

The stagnation pressure during MHD electrical power generation stage decreases because of the dissipation accompanying processes. These stagnation pressure losses can be expressed with earlier defined efficiencies as following

\[
P_{stag,D}^{*} = P_{stag,MHD} = P_{stag,D} \left( 1 - \eta_{e,MHD} \right) \frac{\gamma}{\gamma - 1}.
\]

Thus, the MHD generator modifies the inlet parameters of the combustor to \( T_{stag,D}^{*} \) and \( P_{stag,D}^{*} \). Substituting these values in formulas of the preceding section one can obtain

\[
\varepsilon_{comb}^{*} = \eta_{i,MHD} \frac{1}{1 + \frac{\gamma - 1}{2} \left( \frac{M_D^2 \eta_D}{1 - \eta_{e,MHD}} \right) \left( 1 - \frac{\gamma - 1}{\gamma M_D^2} \right) \left( 1 - \frac{T_{stag,D}}{T_{limit}} \right) \left( 1 - \eta_{e,MHD} \right)^{-1}}.
\]

As it was indicated above the final stage of the AJAX type cycle is an injection of (probably some fraction of) electrical power extracted in upstream section of such a device. The fractioning of the extracted power is connected with energy pre-ionization needs. With these definitions one can get

\[
T_{stag,MHD} = T_{limit} + \eta_{e,MHD} \left( 1 - \eta_{pre-ionization} \right) T_{stag,D} = T_{limit} \left( 1 + \Lambda_{MHD} \right),
\]

where

\[
\Lambda_{MHD} = \frac{T_{stag,D}}{T_{limit}} \eta_{e,MHD} \left( 1 - \eta_{pre-ionization} \right)
\]
and

\[
\eta_{pre-ionization} = q_{e,MHD} \frac{q_{pre-ionization}}{1 - \eta_{pre-ionization}} \leq 1.
\]

Then the effective stagnation pressure available in nozzle expansion process can calculated with

\[
P_{stag,MHDacc} = P_{stag,MHD} \left( 1 + \Lambda_{MHD} \right) \eta_{i,MHDacc} \frac{1}{1 - \eta_{e,MHDacc}}
\]
and the corresponding value of the expansion factor in the nozzle

\[
\varepsilon_{acc,MHDacc} = \left( \frac{P_a}{P_{stag,MHDacc}} \right)^{\gamma - 1} = \frac{1}{\left( 1 + \Lambda_{MHD} \right) \eta_{i,MHDacc}} \eta_{comb}^{\gamma - 1}.
\]

These yield the exhaust velocity

\[
V_{ex} = \sqrt{2 \gamma \left( 1 + \Lambda_{MHD} \right) \left( 1 - \varepsilon_{acc,MHDacc} \right)}
\]
and the thrust

\[
T_{AJAX} = m_\alpha V_a \left( \frac{V_{ex}}{V_a} \left( 1 + f \right) - 1 \right).
\]

3.6. T-S diagram of the AJAX type cycle

The calculation performed in the previous sections are to be qualitatively explained with a simplified T-S diagram analysis. In Fig.4 the similar cycle analyses is shown corresponding to AJAX type concept from the Fig.5 and Fig.6. The latter represents the schematic gram of MHD augmented
scramjet with characteristics points indicated along the slow train.

**Fig.4**

It can be shown also that the final result of MHD augmentations is very sensitive to non-isentropic losses along the MHD processes both in generator and accelerator models.

**Fig.5**

**Fig.6**

**Conclusion**

The analysis carried out in two study is has shown that the MHD augmented jet propulsion system can be provided in frame of existing approves wever the significant improvement of scramjet characteristics is limited by the rather high level of non-isentropic losses typical of MHD systems.

At the same time the possible MHD applications can be found to extend the Mach number range of scramjet operation.

**References**


