5. T-LAYER MHD IN AEROSPACE APPLICATIONS

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Abstract. The problem of electricity production on board of the hypersonic airplane, is considered can be solved applying the MHD method of energy conversion, that allows to converse the kinetic energy of incident flow into the electricity directly.

For these proposes the MHD generator with T-layers is proposed to be used.

Numerical simulation of the MHD process in the generator channel has shown that in the given conditions the regime with multi-structure of T-layers can be organized.

The new type of electric rocket engine is also proposed, using the T-layer effect. As the T-layer is a product of the overheating instability that at the nonlinear phase leads to forming of stable plasma piston (T-layer), artificial creation of T-layers in the flow of working gas allows to suppress the catastrophic development of instability in the whole flow. Numerical simulation of the corresponding process has shown that effective processes of acceleration of nonuniform gas-plasma flow in the channel of MHD accelerator can be organized.

The ionization kinetics of nonequilibrium current layers (T-layers) in the Faraday channel of closed cycle MHD generator has been investigated. Various noble gases (helium, neon, argon) not containing alkali seed have been considered as a working body of such generator. The calculations have shown that the optimal characteristics of generator process are achieved in the neon flow with stagnation pressure 2 MPa, stagnation temperature 2000 K and maximum value of magnetic field 10 T. The calculated efficiency of MHD conversion has amounted 41% for the enthalpy extraction ratio and 79% for the adiabatic efficiency.

I. AIRBORNE MHD GENERATOR FOR A HYPERSONIC AIRPLANE

It is well known that the main problem of application of MHD generators aboard an airplane is bulky and heavy magnetic systems. Traditional MHD method of energy conversion deals with very low efficient electric conductivity of a uniform flow of a working medium. In order to adjust this drawback according to the formula for specific power of MHD generator

\[ W = \sigma K (1 - K) \mu B^2 \]

it is necessary to increase magnetic field. The required value \((B \approx 4T)\) is realized in superconducting electromagnets whose weight and size characteristics are not suitable for airplane conditions.

In a MHD generator with T-layers there is formed a non-uniform stream where T-layer occupies no more than 10% of the MHD channel volume. Plasma electric conductivity in current layers \(\geq 10^3 S/m\) and under these conditions average electric conductivity of the whole flow considerably exceeds a uniform case. Therefore, it is possible to considerably decrease magnetic field to the parameters \((B \leq 1T)\) which are realized by means of permanent magnets.

A MHD generator on a hypersonic airplane converting kinetic energy of a running on air flow into electric energy won’t make on very strict demands to isentropic efficiency of the process. Hence one can decrease a load factor here and turn to maximum specific power mode in the MHD channel, that is to work with \(K = 0.5\). In its turn it changes relation of factors: Joule dissipation \(j^2/\sigma\) and force \(j \times B\). Now execution of conditions of plasma self-maintenance won’t be followed by extremely strong magnetohydrodynamic interaction of T-layers and a flow which enables to reduce losses in shock waves arising in the course of such interaction. Finally, work with air and not with combustion products enables to reduce radiation losses which changes the structure of a T-layer. It becomes thicker and accordingly more stable. Summing up all these, one should acknowledge that the conclusion on uselessness of a MHD generator with T-layers for work in open cycle [1] has proved to be wrong for a hypersonic airplane conditions. It will probably turn out to be the most efficient facility as an airborne source of electric energy.
The value of magnetic field should be selected in order to find an optimal mode.

Mathematical model.

The process will be described by the system of quasi-one-dimensional equations of magnetohydrodynamics

\[
\frac{\partial \rho A}{\partial t} + \frac{\partial \rho u A}{\partial x} = 0
\]  (4)

\[
\frac{\partial \rho u A}{\partial t} + \frac{\partial \left(\rho u^2 + P\right) A}{\partial x} - P\frac{\partial A}{\partial x} = jBA
\]  (5)

\[
\frac{\partial e A}{\partial t} + \frac{\partial \left(euA + P\frac{\partial A}{\partial x}\right)}{\partial x} = (jE - q_e)A
\]  (6)

Here \(A(x)\) - cross-section of the channel, \(e = e_0 + u^2/2\) — complete gas energy, \(q_e\) — power of radiation losses of energy from a unit of volume. For current density and electric field the following equations are used:

\[j = \sigma(E - uB), \quad E = K uB\]  (7)

Equation system (4) – (6) is complemented with thermodynamic relationships for pressure \(P(p, e)\) and temperature \(T(p, e)\) given in tables and dependence on them of electric conductivity \(\sigma(T, P)\). Volume radiation losses for air can be estimated by a model of a volume radiator

\[q_e = 4\sigma_\infty T^4 \left[ \frac{e_r(P, T, x - x_1)}{(x - x_1)} + \frac{e_r(P, T, x_2 - x)}{(x_2 - x)} \right]\]  (8)

Here \(\sigma_\infty = 5.67 \times 10^{-8} \text{W/m}^2\text{K}^4\); \(x_1 < x < x_2\), where \(x_1\) and \(x_2\) are distances from the calculated point up to the left and right boundaries of a T-layer correspondingly; \(e_r(P, T, l)\) is radiation ability of hemispheric isothermal volume with radius \(l\). Relationship (8) was checked in correlation with more accurate calculation [2] for a non-isothermal layer of air plasma. In a wide range of changes \(P, T\) and \(l\) deviation did not exceed 50%.

Boundary conditions of the problem are determined by its formulation: at the MHD channel input (cross-section \(x=0\)) stationary conditions with the parameters: \(P_0 = 1.13 \times 10^5 \text{Pa}; \quad T_0 = 925 \text{K}; \quad u_0 = 1796 \text{m/s}\) are being set. At the channel output \((x=6m)\) the flow has a pulsating nature but still remains a super-sonic one which allows to prescribe conditions \(\partial \phi/\partial x = 0\). Steady isentropic flow is an initial condition of the problem.

For numerical solution of the (4)-(6) set of equations the two-step difference Lax-

<table>
<thead>
<tr>
<th>Parameters of the simulated airborne MHD generator</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Static Pressure in a running on flow</td>
<td>10^5 Pa</td>
</tr>
<tr>
<td>2. Air temperature overboard</td>
<td>240K</td>
</tr>
<tr>
<td>3. Flight speed</td>
<td>2135 m/s</td>
</tr>
<tr>
<td>4. Pressure in the throat of the channel (M=3)</td>
<td>1.13\times10^5 Pa</td>
</tr>
<tr>
<td>5. Temperature in the throat of the channel</td>
<td>925 K</td>
</tr>
<tr>
<td>6. Speed of air in the throat of the channel</td>
<td>1796 m/s</td>
</tr>
<tr>
<td>7. Cross-section of the throat</td>
<td>57\times10^{-4} m^2</td>
</tr>
<tr>
<td>8. Width of electrodes</td>
<td>0.1 m</td>
</tr>
<tr>
<td>9. Height of the channel in the throat</td>
<td>0.057 m</td>
</tr>
<tr>
<td>10. Height of the channel in the output</td>
<td>1 m</td>
</tr>
<tr>
<td>11. Initial temperature of a plasmoid</td>
<td>7\times10^7 K</td>
</tr>
<tr>
<td>12. Initial thickness of a plasmoid</td>
<td>0.05 m</td>
</tr>
<tr>
<td>13. Length of the MHD channel</td>
<td>6 m</td>
</tr>
<tr>
<td>14. Load factor</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Vendroff scheme is used modified by means of the local dissipation method.

Results of simulation.

In Fig.2 one can see distribution of speed, pressure and temperature in a MHD generator channel on one of the moments of steady-state periodic process. Steady distribution of magnetic field shown in Fig.3 corresponds to the given mode. In this moment of time there are five current layers (see relationship T(x)) in the MHD generator channel and their maximum temperature reaches $12 \times 10^3 \text{K}$. The current layers actively interact with the flow which results in arising of shock waves and expansion waves. These waves can be well identified on the curves $P(x)$ and $u(x)$, the gradient zones with $\partial P/\partial x > 0$ corresponding to the fronts of shock waves and zones with $\partial U/\partial x < 0$ being current layers where overfall of pressure is balanced by electrodynamic force.

![Fig. 2](image)

![Fig. 3](image)

In development of T-layers one can underline the following stages:

1. Stage of "picking up" of temperature disturbance set at the MHD channel input. In the flow of air not containing alkali seed at the initial temperature about 7000K disturbance has low electric conductivity and is moved by the flow practically without force interaction. However, in a short-circuit section of the channel the power of Joule dissipation in plasma exceeds cooling due to expansion of the flow. Then the mechanism of overheating instability turns on which results in explosive-like dynamics of the process. Pressure in a T-layer does not have time to level off and, as seen from the graph $P(x)$ for current layer turned out in the cross-section $x=1 \text{m}$, the pressure peak corresponds to the thermal spike. Temperature of the T-layer at that moment of time reaches 12000K and its farther elevation is decelerated by energy losses to radiation.

2. Working stage. T-layer is thermally stabilized by radiation loss and temperature in it does not rise. The pressure peak decays and instead of it there arises the pressure gradient balancing electrodynamic force. At this stage the flow acting upon a T-layer as if on a plasma piston does work a part of which is released in the load and the remained part maintains self-support mode of a T-layer in the form of Joule dissipation. As the T-layer travels along the channel, gas pressure is continuously dropping and, hence, the power of radiation losses is dropping too. Limitation of unwanted temperature rise is attained by means of decreasing of magnetic field along the channel. Efficient periodic mode of operation with five current layers in the working section of the MHD channel is supported by magnetic field with average value $<B>=0.3 \text{T}$. Such a field is easily created by permanent magnets.

In the process of numerical simulation electric power of a MHD generator was being determined which made the value $N=3 \text{MW}$ in the periodic mode. Expenditure for own needs, i.e. for initiating current layers made $N_{\text{exp}}=0.5 \text{MW}$ in this investigation. This is net expenditure ignoring losses which in real discharge systems can make up to 50% but even in this case there remains a considerable store of useful power which makes the idea of the MHD generator with T-layers attractive for the given applied utilization.
II. EFFICIENCY ANALYSIS OF THE SPACE MHD POWER PLANT

Efficiency of a power plant, as a rule, is associated with the high value of thermal efficiency that defines the transforming ratio of the heat energy received from a heat source. Thermal efficiency of any heat machine can be conceived of as a product

$$\eta = \eta_{\text{Carnot}} \eta_{\text{Perfect}},$$

where $\eta_{\text{Carnot}} = (T_{\text{max}} - T_{\text{min}})/T_{\text{max}}$ is the efficiency of the Carnot cycle carrying out isothermal heating of the working body at temperature of a heat source $T_{\text{max}}$ and isothermal cooling at the temperature of a refrigerator $T_{\text{min}}$; $\eta_{\text{Perfect}}$ is a parameter reflecting thermodynamic perfection of the concrete power plant. The meaning of the second factor is defined by the irreversibility degree of heat exchange processes which is proportional to the temperature difference between heat exchange mediums. The temperature rise $T_{\text{max}}$ does not have to be accompanied of the general thermal efficiency upgrowth. So for example the transition from steam-gas installation with $T_{\text{max}} = 1500K$ to the MHD generator with $T_{\text{max}} = 3000K$ at common $T_{\text{min}} = 300K$ increases $\eta_{\text{Carno}}$ from 80% up to 90%, i.e. only by 10%. As this takes place the gas turbine has at the output the temperature about 900K and the MHD generator about 2300K while the bottom cycle of the steam turbine has in both cases $T_{\text{max}}$ about 800-900K. By inevitable losses, which appear because of the irreversible heat exchange processes, the MHD generator will lose to steam-gas installation in thermal efficiency in earthly conditions ($T_{\text{min}} = 300K$).

To reveal the most effective type of a space power plant it is necessary to carry out the elementary thermodynamic analysis of possible cycles of heat machines. We shall consider gas turbines and MHD generators for which the heat source heats up a working body without phase transitions that much simplifies a heater design. In space conditions it can be of paramount importance. Clearly in space only a closed cycle installation may be used having a heat source and device carrying out a working adiabatic process, together with the refrigerator and compressor restoring the gas pressure. To reduce the compressor work it is necessary with the help of refrigerator to down the temperature $T_{\text{min}}$ so far as possible. In space conditions cooling of the working body in a refrigerator can be carried out only by dispersing heat energy in the form of thermal radiation. The power of this radiation from a unit of surface is proportional to the fourth degree of temperature. Hence, decrease of the surface temperature of refrigerator radiating panels will result in sharp growth of their surface, and with it the growth of the total installation weight. On the other hand, temperature rise leads to reduction of thermal efficiency of installation. In these conditions there is an optimum temperature of a refrigerator which provides the maximum value of specific power. The thermodynamic analysis of power plant cycles that are based on the gas turbine and MHD generator given below, permits to define thermal efficiency of installations, their specific power and, in the end, to find the value of the optimum characteristics. We shall consider that the structure of installations corresponds to the scheme shown on Fig.4 where the block "converter" represents either a gas turbine with the electrogenerator or a MHD generator. This scheme is simple enough but at the same time its efficiency is close to the limiting value achievable for this type installations.

The appropriate thermodynamic cycle of these installations is shown on Fig. 5.
The thermal efficiency of the given installations will be defined as the relation
\[ \eta = \frac{W_{\text{gen}} - W_{\text{com}}}{Q_{\text{heat}}}, \]  
where \( W_{\text{gen}} \) - the power of a heat energy converter, \( W_{\text{com}} \) - the power of a compressor, and \( Q_{\text{heat}} \) - heat power of a heat source. 

A working body of a closed cycle will be noble gas which is on the one hand a good heat-carrier, and on the other hand, because of its chemical inertness, does not cause chemical erosion of the channel walls. For noble gas the specific heat could be conceded as a constant value and, hence, the parameters in the formula (9) can be expressed as
\[ W_{\text{gen}} = G \tilde{C}_p (T_1 - T_2), \]
\[ W_{\text{com}} = 3 \tilde{G} \tilde{C}_p (T_3 - T_4), \]
\[ Q_{\text{heat}} = G \tilde{C}_p (T_1 - T_{10}). \]

Therefore, the ratio (9) is written down as:
\[ \eta = \frac{T_1 - T_2 - 3(T_3 - T_4)}{T_1 - T_{10}} \]  

(10)

Here the temperatures \( T_1 \) and \( T_2 \), being the temperatures \( T_{\text{max}} \) and \( T_{\text{min}} \), are defined accordingly by conditions in a heat source and in the refrigerator. In this analysis value of \( T_{\text{max}} \) is set which is 1200K for the gas turbine and is 2500K for the MHD generator. The value of \( T_{\text{min}} \) will vary for search of an optimum value. The value of the temperature \( T_2 \) is defined by work conditions of the heat energy converter in which an irreversible adiabatic process proceeds. Let there be given the ratio of the inlet pressure to the outlet one as a parameter \( E = P_1/P_2 \) and the parameter of adiabatic efficiency \( \eta_s = (T_1 - T_2)/(T_1 - T_2) \) which is defined as the ratio of a real difference of temperatures to an ideal isentropic one. Then the temperature \( T_2 \) is defined as
\[ T_2 = T_1 \left[ 1 - \frac{1}{\eta_s} \left( 1 - \beta^{-(y-1)/\gamma} \right) \right] \]  

(11)

For a compressor step the ratio of compression is \( \pi = \beta^{1/3} \) and adiabatic efficiency is \( \eta_{sk} = (T_3 - T_4)/(T_3 - T_4) \) therefore the temperature \( T_3 \) at the compressor step outlet is defined as
\[ T_3 = T_4 \left[ 1 + \frac{1}{\eta_{sk}} \left( \beta^{(y-1)/3y} - 1 \right) \right] \]  

(12)

The temperature at the heat source inlet is defined by the temperature difference on a wall of a tubular heat exchanger which is used for utilization of the heat energy of the working body after the converter. As a rule, this difference \( \Delta T \) is set equal 50K. Thus the of temperature value at the heat source inlet will be defined as \( T_{10} = T_2 + \Delta T \).

Finally, the formula for calculation of thermal efficiency will look like
\[ \eta = \frac{T_1 \left[ \frac{1}{\eta_{sk}} \left( \frac{1}{1 - \beta^{-(y-1)/\gamma}} \right) - 3 \frac{1}{\eta_s} \left( \beta^{(y-1)/3y} - 1 \right) \right]}{T_1 \left[ \frac{1}{\eta_{sk}} \left( \frac{1}{1 - \beta^{-(y-1)/\gamma}} \right) \right] - \Delta T} \]  

(13)

Let us use this formula and define the dependence \( \eta(T_4, \beta) \), thus hitherto not determined parameters in the formula (13) will have the values:
\( \eta_s = 0.7 \) for the MHD generator and 0.9 for the gas turbine;
\( \eta_{sk} = 0.87 \) is assimilated level for the axial compressors.

The results of the calculation are submitted on a Fig. 6 where (A) corresponds to the conditions of a MHD generator and (b) those of a gas turbine. The area of parameters for which the thermal efficiency has appeared to be less than zero is given by zero value on diagrams.

In conditions of space the value of thermal efficiency itself is not of prime importance. Here, as stated above, the determining parameter \( \varphi \) is the specific power calculated as the ratio of electrical power to the total weight of installation. For calculation of this parameter it is necessary to estimate the weight of all units making up the power plant. Let us use the data [3] that analyzes a nuclear space power plant...
with open cycle disk MHD generator created on the basis of the space nuclear rocket engine (the so-called project NERVA with high-temperature gas-cooling nuclear reactor). The reactor parameters: heat power \( \approx 300\text{MW} \); the heat-carrier - hydrogen; temperature of heating \( \approx 2500\text{K} \); pressure of hydrogen \( \approx 30\text{atm} \). This reactor, apparently, can be altered for noble gas with preservation of a former temperature level. The mass characteristics this installation are the following:

1. Nuclear reactor 2200kg
2. MHD generator 2100kg
3. Superconducting electromagnet 1500kg

In addition to this list the closed cycle installation should include a further compressor group with electric motors and system of start, tubular heat exchanger, radiating refrigerator, and besides, for the gas turbine the installation should contain the turbine generator instead of the MHD generator and electromagnet. The weight of these elements can be estimated only by convention. Let us consider, that for a given heat power all additional units, except a refrigerator, have the same weight of 2000kg and the weight of the turbine generator is equal to the weight of the MHD generator and the electromagnet. Thus, we establish the following conformity of weights:

4. Compressor group 2000kg
5. Electric motors and system of start 2000kg
6. Tubular heat exchanger 2000kg
7. Turbine generator 3600kg

Weight of a radiating refrigerator should depend on the dispersed heat power. Let us consider the weight of one square meter surface of heat-dispersing panels equal 1 kg, and flow of energy from a surface appropriate to the radiation law of the black body. In this case the weight of the refrigerator is \( M_{\text{ref}} = Q_{\text{heat}}/(\sigma \cdot T_{4}^{4}) \)kg where \( Q_{\text{heat}} = 3 \times 10^{8}\text{W} \) and parameter \( \sigma \) - is Stefan’s constant \( = 5.67 \times 10^{-8}\text{W/(m}^{2}\text{K}^{4}) \).

Finally the specific power of installation is defined as

\[ \varphi = Q_{\text{heat}} \eta / M, \text{ where } M = 11800 + M_{\text{ref}} \text{kg}. \]

Fig.7 shows the results of specific power calculation for installations with the MHD generator (Fig.7a) and with the gas turbine (Fig.7b) as functions of parameters \( \beta \) and \( T_{4} \). It is clear that the specific power really has optimum value in both cases, for the MHD generator it being reached at \( T_{4} \approx 600\text{K} \) and for the gas turbine at \( \approx 350\text{K} \). For technical reasons the heat power circulating into the installation should be comparable with the generated electrical power that implies the necessity to use the installations with a high value of enthalpy extraction. Let us accept for optimum value \( \beta = P_{1}/P_{2} = 10 \) that at \( \eta = 0.7 \) corresponds to the enthalpy extraction ratio

\[ \eta_{N} = \eta_{0}(1 - \beta^{-\gamma})/\gamma = 0.4 = 0.4 \] (14)

At these parameters the specific power of the MHD generator makes \( 2000\text{W/kg} \) and that of the gas turbine about \( 300\text{W/kg} \). Since, there is a large uncertainty degree in the definition of the weight characteristics, this result should be estimated as a qualitative one which demonstrates that the gas turbine and solar photo cells have identical levels of efficiency and the MHD generator can lift efficiency of the space power plant to an order.
For the first time the problem of creation a MHD accelerator using T-layers was analyzed in the work [4] considering the mechanism of gas flow acceleration by a separately taken T-layer. As this takes place, the channel of the accelerator had a constant section that interfered with effective work of electrodynamics force. In work [4] a phenomenon of acceleration is fixed, though the additional gain of velocity appeared to be small. In this research the problem of acceleration is

\[ M = 1 \text{m} \]

the average gas acceleration is \( a = 2 \cdot 10^7 \text{g/s} \). If the induction of magnetic field equals 10T then from a ratio \( \rho \cdot a=j \cdot B \), in which average mass density of the flow is \( \rho = 0.1 \text{kg/m}^3 \), find the average current density as \( j \approx 2 \cdot 10^5 \text{A/m}^2 \). Current density in the T-layers is an order higher, i.e. \( j_{\text{max}} \approx 200 \text{A/cm}^2 \), that corresponds to a current density in the electrical rocket propulsion more than 100 times exceeds the initial enthalpy. At the length of MHD channel equal 1m the average gas acceleration equals \( a = 10^7 \text{m/s}^2 \). For function of radiating losses a simplified model is accepted \( Q_r = Q_0 \, T^\gamma \) where \( Q_0 \) is the loss of energy from plasma with radiation \( \text{W/m}^3 \); \( \gamma \) - specific energy of the erosion products \( \text{J/kg} \).

A current density is defined from the Ohm’s law \( j = \sigma(E - uB) \) where \( B=10 \text{T} = \text{Const} \), and the intensity of the electric field is set as a function

\[ E = E_0(1 + t/T_0)(1 + x/X_0) \quad (15) \]

The parameters \( T_0 \) and \( X_0 \) have been selected so as to ensure the growth of electric field in the first \( 10^{-5} \)s after the start \( T_0 = t \) for \( \pm 10^{-5} \)s and continuous linear growth of the field on the length of the MHD accelerator.

The plasma conductivity is set as a function of gas temperature and pressure in conformity with the model for noble gas [5]. For function of radiating losses a simplified model is accepted \( Q_r(T) = Q_0 \, T^\gamma \) where factor \( Q_0 = 10^7 \text{W/m}^3 \text{K}^{-4} \) is selected so as the stabilizaton of a T-layer occurred at the temperature \( \pm 20000 \text{K} \). The source of mass in the equation of mass conservation is defined in conformity with experimental data about the arc erosion on copper electrodes which can be summarized in the form of a ratio between the vaporized mass and the amount of electricity gone through the arc, \( K_q = 2 \times 10^4 \text{kg/C} \). Let define...
a factor of proportionality between current density and the right part of the mass conservation equation with condition \( MA=K_q \) where \( a \) is the width of electrodes =0,01m.

The initial condition of the problem is a stationary isentropic flow which does not interact with magnetic field. The non-stationary process begins with the task isobaric perturbation of the temperature on an inlet site of the accelerator channel. The electrodynamics force operating in a volume of plasma perturbation causes its acceleration that is accompanied by formation of the waves of both compression and divergence. The wave of divergence moves to the up flow with a sound velocity but the supersonic flow maintained at the channel inlet keeps stationary conditions here that can be used as the boundary conditions. A boundary condition at the outlet section of the channel are the conditions of "transference", i.e. condition of equality to zero derivative on \( x \) for all parameters of flow.

Thus in this section the statement of the problem of numerical simulation of the MHD process in the channel of the electrical rocket propulsion is formulated. The problem was decided with the use of algorithm constructed on the basis of the Lax-Wendroff’s method which was used at the simulation of the MHD generator processes.

**Simulation results of the MHD process in the ERP channel**

On Fig.8 the acceleration process of the flow in the form of dependencies: \( u(x) \), \( \rho(x) \), \( T(x) \) is submitted. Specially selected electric field dynamics in the channel corresponds to this mode (see dependence \( E(x,t) \) on Fig.9), which includes the increasing of \( E \) both temporal and spatial in the working part of the channel according to a condition (15), but also temporary growth of value \( E_0 \) which defines an electrical field in the formation section of the T-layers. This growth appeared to be necessary for formation of new T-layers following the first one.

The new layers appear in the zone of a rarefied wave generated by the first T-layer. As a result they receive additional acceleration not connected to the action of electrodynamics force. The growth of velocity in this case reduces the total electric field \( E*=E_0+aB \) which without increase of an external field \( E \) is not sufficient for formation of selfmaintained current layer.

**Fig. 8**

Presented on Fig.8 distributions of the main gasdynamic parameters correspond to the established periodic mode. From this figure we notice that the average flow velocity grows linearly and on the channel length per 1m the velocity about 16km/s is reached. The flow of the working body gets a pulsing character that is well visible from distribution of pressure in the channel of the accelerator. Thus the T-layer are placed in the sites of the flow appropriate to the zones with minimum density which are formed.

**Fig. 9**

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under the influence of two factors: thermal expansion of the gas heated up in a T-layer and action of a rarefied wave. Density of plasma in T-layers is on average two orders of magnitude lower than in the surrounding gas flow that leads to the development of overheating instability. As a result the current layers become thinner (1-2cm instead of 5-7cm of initial perturbation) and its temperature is stabilized at the level $a_{10^4K}$. Conductivity of plasma reaches the values about $10^{-4}$S/m at which for maintenance of a T-layer, as differentiated from the conditions of the formation, very small value of the field $E^*$ is required. On Fig.10 the distribution of $E^* (x)$, appropriate to the same time moment as on Fig.8 is given. On this curve it is possible to see a series of "holes" which correspond to the zones of T-layers. At the inlet site of the channel, where the mode of T-layer formation takes place, the minimum value of $E^*$ in the "holes" is about 10V/cm while in the other part of the channel these minimums are practically about zero.

**Fig. 10**

Therefore, the autoregulation takes place at which changing the electric field it is possible to set a velocity of T-layers and through it the flow velocity too. On the channel length about 1m the acceleration process proceeds in a regular mode that allows to increase the velocity of a flow up to the desirable value by simple escalating of the channel length with preservation of linear growth of $E$.

IV. IONIZATION AND RECOMBINATION PROCESSES IN THE NONEQUILIBRIUM PLASMA IN CONDITIONS OF FARADAY CHANNEL OF MHD GENERATOR

**Molecular Ions Kinetics**

A significant part of ions can exist in the molecular form. Molecular ions are generated as a result of the atomic ions conversion with the scheme $A^+ + 2A \leftrightarrow A_2^+ + A$ and in the associative ionization process $A^* + A \leftrightarrow A_2^* + e^-$, where $A^*$ is an atom in the high excited state, which excitation energy differs from the ionization potential by the value close to the molecular ion dissociation energy $E_D=1.4eV$. The process, reverse of the associative ionization is the dissociative recombination of molecular ions and electrons, causes the additional electron losses. At a low gas temperature the equilibrium in these reactions shifts to the conversion of atomic ions into molecular ones and dissociative recombination. The equation of the molecular ions density balance, formed by these reactions, is written as

$$\frac{\partial n_m}{\partial t} = c_1 n_a^2 n_i - c_2 n_m n_a + \beta_m n^* n_a - d_m n_m n_e$$  (16)

The reaction rate constant of atomic ions conversion [9] depends on the gas temperature as $c_1 \sim T^{-3/4}$. At 300K $c_1=4.1 \times 10^{-22}cm^6/s^3$. The ratio of the reverse process is found from the thermodynamic equilibrium:

$$K_m = \frac{g_1 g_2}{g_m} \frac{\exp \left( \frac{E_D}{T_g} \right)}{2 \pi \hbar^2} \sqrt{\frac{\mu g}{2 \pi \hbar}} \exp \left( -\frac{h \nu}{T_g} \right)$$

This expression includes the following characteristics of the molecular ion $Ne_2^+$: statistic weight $g_1=3$; oscillation quantum energy $h \nu=7.7 \times 10^{-2}$eV; dissociation energy $E_D=1.4eV$; distance between nuclei $r_0=1.7A$ [4].

According [5] $d_m = 1.8 \times 10^{-7}T_e^{-0.43}T_g^{1.1}cm^3/s$, as a result of dissociative recombination two atoms appear, one of which is in the base state and another in the $3p, 3p'$ state.

The atomic ion converts into the molecular one during the time of the order of $\tau = c_1 n_a^2 \approx 10^{-6}+10^{-5}s$, which is significantly less than the characteristic diffusion time of ion. That is why the diffusion is not taken into account in the equation of balance of molecular ions density.
Electron-Atom Non-elastic Collisions

In the excited atoms of noble gases due to the high potential of the first excited level there is one electron on the higher shell, weakly bounded to the atomic residual. Due to this the collision transitions between the excited levels can be considered as the diffusion of electrons in the discrete energy space. Solving the Focker-Planck equation for the discrete spectrum \[6,\text{app.2}\] the following expression is obtained for the transition probability from the level \(k\) to \(k+1\), in the case of the maxwellian energy distribution of free electrons:

\[
\frac{\sqrt{m_eT_e}}{E_{k+1}-E_k} \exp\left[-\frac{E_{k+1}-E_k}{E_e}\right] \exp\left[-\frac{E_{k+1}-E_k}{E_e}\right]
\]

(17)

Here \(\Lambda_k\) is the Coulomb logarithm of transition. The reverse processes probabilities are defined from the equilibrium correlation. The electron-electron energy transfer collision frequency is much more than that against electron-heavy particle, i.e. the main body of the electron energy distribution is maxwellian. Due to small values of ratios of electron and atom masses and also electron-atom and coulomb cross sections it becomes true at the ionization ratio \(\approx 10^{-7}\) [6,7,1]. Nonelastic electron-atom collisions with the transitions between the excited levels with the energy \(\approx 2\text{eV}\), only change the temperature of the main body of the distribution function, so we can use the equilibrium values for the probabilities of transitions between the excited levels \(z_{k,k+1}\), when \(k\geq0\): \(z_{k,k+1}=z_{k,k+1}^0\).

It is seen from (17) that closely located levels exchange the electrons very intensively. That is why we can combine closely-spaced levels adding their statistical weights and averaging out the energy [6,app.3]. Lets choose the following effective levels:

<table>
<thead>
<tr>
<th>Combined levels</th>
<th>Average energy (E_k), eV</th>
<th>Total statistical weight (g_k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1S0</td>
<td>-21.56</td>
</tr>
<tr>
<td>1</td>
<td>3s, 3s'</td>
<td>-4.86</td>
</tr>
<tr>
<td>2</td>
<td>3p, 3p'</td>
<td>-2.94</td>
</tr>
<tr>
<td>3</td>
<td>4s, 4s'</td>
<td>-1.87</td>
</tr>
<tr>
<td>4</td>
<td>4p, 4p', 3d, 3d</td>
<td>-1.46</td>
</tr>
</tbody>
</table>

The fourth effective level includes the states which ionization energy is close to the dissociation energy of the molecular ion; in the equation (16) \(n^*\approx n_0\).

To find the cross section of excitation from the base state the half-empiric formula is used [7]:

\[
q_{01}(u)=\frac{n_e^4 f_{01}}{(E_1-E_0)^2} \left(\frac{u}{u+1}\right)^{0.3} \ln(1.25(u+1))
\]

(18)

where \(u=(E-E_1+E_0)/(E_1-E_0)\). With the approach \(u=(E_1-E_0)/T_e\ll1\) the frequency of the excitation from the base state

\[
0 = \frac{2n_e^4 f_{01}}{(E_1-E_0)^2} \exp(-t) \left(\frac{\ln(1.25)}{t} + 0.3\right)
\]

(19)

The collisions with the excitation from the base state strongly influence the high energy part of electron energy distribution. Let us estimate the derivation of the electron energy distribution from the Maxwellian, following [6.7.2.2]. The estimation expression obtained there, is

\[
z_{01}(t)=\frac{2n_e^4 f_{01}}{(E_1-E_0)^2} \exp(-t) \left(\frac{\ln(1.25)}{t} + 0.3\right)
\]

Ionization from the low excited states can be a significant source of electrons. The following half-empiric formula is derived in [7] for the cross section of ionization from the \(k\) level:

\[
r_k = \frac{n_e^4 f_k N}{E_k} \frac{1}{u^2} \ln(1.25u)
\]

(21)

where \(N\) — the number of equivalent electrons in the atomic shell (for the excited states of noble gases \(N=1\)), \(f_k\) — oscillator strength. The ionization rate

\[
\beta_k(t)=\frac{2n_e^4 f_k N}{E_k^2} \frac{1}{m_eT_e} \exp(-t)
\]

(22)

The ionization from the 4th level and three-body recombination to the 4th level are described within the framework of the diffusional approach, giving for the three-body recombination coefficient the expression called the “9/2 law”:

\[
\alpha_d=\frac{4\sqrt{2\pi}}{9} \frac{e^{10\Lambda}}{m_eT_e^{3/2}}
\]

(23)

The ionization coefficient is found from the equilibrium correlation.
Radiation Transitions

Let us consider that in a dense plasma, with the parameters like the described above, the radiation transitions to the base state are suppressed by the reabsorbing of radiation. At the same time we must take into account the atom impact quenching of the resonance state that strongly influences the populations of low excited levels, in the case of low ionization. The opposite situation is observed in the case of transitions between the excited levels, where the radiation transitions prevail over the atom impact quenching. In the nonequilibrium plasma under the given conditions the density of the excited atoms amounts to $10^8$–$10^{10}$cm$^{-3}$. That is why we can consider the plasma to be transparent for the radiation of transitions between the excited levels. As approaching the continuous spectrum the influence of radiation processes, as compared with the processes of electron impact excitation and quenching, sharply decreases. The estimations of the collision and radiation transitions intensity [6.5.1] show that the influence of radiation processes becomes negligibly small at the level energy $\frac{\hbar c}{\epsilon} = 1.8 \text{ eV cm}^2$. The experimental data about its value lies within the limits $10^{-19}$–$10^{-20}$cm$^2$/eV. This value strongly influences the discharge model characteristics. That is why this value was chosen so that the discharge model were most close to the real characteristics. The optimal value appeared to be $q_{\text{at}} = 8.7 \times 10^{-20}$cm$^2$/eV

The Kinetic Equations System

The populations of four effective levels and the density of the molecular ions are described by the following system:

$$z_{01}^0 F_{n_a} - (z_{01}^0 F + Q_{10}^0 + \beta_1) n_1 + (z_{21} + A_1) m_2 = 0$$

$$z_{12}^1 n_1 - (z_{21} + A_1 + \beta_2) m_2 + z_{32} n_3 + A_{42} n_4 + + d_m n_e n_m = 0$$

$$z_{23}^2 n_2 - (z_{32} + \beta_3) n_3 + z_{43} n_4 = 0$$

$$z_{34} n_3 - (z_{43} + \beta_4 + \beta_m n_a) n_4 + a_3 n_e^2 (n_e - n_m) = 0$$

$$c_i n_e^2 (n_e - n_m) - z_{2} n_e n_m - \beta d_4 n_4 - d_m n_e n_m = 0$$

The schematic view of this system is presented on Fig. 11.

![Fig. 11. Scheme of fluxes in the discrete energy space. Simple arrows — the electron impact transitions, wavy arrows — the radiation transitions, thick arrows — the atom impact transitions and processes with participation of molecular ions](image)

The system is linear relatively to $n_1$, $n_4$ and $n_m$. As a result of its solution we obtain the dependency of these values on $n_e$, $n_a$, $T_e$ and $T_g$. The speed of electrons delivery or the electron flux in the energetic space is found from the expression

$$J_E = z_{01}^0 F_{n_a} - z_{10}^0 F_{n_1} - Q_{10}^1 n_1$$

The electron gas energy losses in the line radiation and with the decay of the molecular ions are described by the expression

$$q = (E_1 - E_0) Q_{10} n_1 + (E_2 - E_1) A_{21} n_2 + (E_3 - E_2) A_{32} n_3 + (3/2 T_e - E_4) d_m n_m n_e$$

Results of simulation of MHD process

To organize the working regime in which the plasma clots are slowly recombinated it is necessary to set a high ionization in them at the beginning. Besides, the gas in the ionized layer must be heated because in cold gas the dissociative recombination is strong and phenomenon of “frozen” ionization is impossible. The task of creation of the plasma clot with needed parameters is the object of
further investigations. Apparently, the plasma clot with the needed ionization and gas temperature may be created by the means of high current diffusive discharge. This type of discharge, described in 8, develops in a strong electric field. It remains diffuse during the characteristic time of gas rarefaction. In the same period electron density grows sharply.

The expense of energy to the plasma clot creation, which consists of the ionization and gas heating expenses, must be deducted from the thermal efficiency of the MHD generator. The working ionization ratio is less than $10^{-4}$ and, since the plasma is about the tenth part of the gas in the MHD channel, the electron gas energy is about $10^{-3}$ of the gas energy. Thus we can neglect the ionization expense. The expense for gas heating was estimated as the heat delivered in the equilibrium isobaric process. The simulation was conducted with the following parameters of flow and MHD-channel:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stagnation pressure</td>
<td>2 Mpa</td>
</tr>
<tr>
<td>Stagnation temperature</td>
<td>2500 K</td>
</tr>
<tr>
<td>Mach number at the inlet</td>
<td>1.5</td>
</tr>
<tr>
<td>Channel length</td>
<td>2 m</td>
</tr>
<tr>
<td>Channel expansion ratio</td>
<td>10</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>10 T</td>
</tr>
<tr>
<td>Load factor</td>
<td>0.7 (at the inlet) \div 0.97 (at the outlet)</td>
</tr>
<tr>
<td>Initial electron density</td>
<td>$7 \times 10^{14}$ cm$^{-3}$</td>
</tr>
<tr>
<td>Initial gas temperature in the T-layer</td>
<td>2500 K</td>
</tr>
<tr>
<td>Periodicity of the T-layers forming</td>
<td>1.8$\times 10^{-4}$ s</td>
</tr>
</tbody>
</table>

With time the periodic regime of flow establishes in the MHD channel. Its character is shown on fig.12,13. Interaction between the plasma layers leads to the deceleration of flow as a whole. The obtained characteristics of the MHD generator efficiency are the following: enthalpy extraction ratio $\eta_H=0.412$; adiabatic efficiency $\eta_s=0.788$. The expenses on the plasma layer creation amount 0.029 of the whole heat power. The working regime by the means of the load factor varying was chosen such that the plasma clots are slowly recombinating, as seen on Fig.14. The situation is observed in which the both ionization and recombination are weak in T-layers during the work time. In this conditions the ionization ratio keeps practically constant. It leads to that the conductivity of plasma decreases with the increase of the electron temperature, which suppresses ionization and overheating instabilities.

Fig. 12. Gas temperature in the isentropic and established flow

Fig. 13. Flow velocity in the isentropic and established flow

Fig. 14. Momentary distribution of ionization ratio in the established mode
Conclusion

As conclusions let us enumerate the main results:

1. The program of industrial exploitation of space will require the creation of a powerful electrical rocket propulsion with thrust force up to 1000N. To feed such an engine aboard of a spacecraft we need a source of energy with the power level up to 10MW. The attractive decision of this problem can be a combined installation including both a source of electric power on the basis of a closed cycle MHD generator and a propulsion on the basis of the MHD accelerator.

2. Comparison of the efficiency has shown that in space conditions the MHD generator is about an order superior both to the gas turbine and the perspective solar cells in specific power (W/kg).

3. As shown, the use of a MHD effect of the formation of self-maintained high-temperature current layers (T-layers), which are electrical arcs stabilized by the losses of radiation, allows to realize in the MHD channel a mode of quasi-continuous acceleration of the gas flow. For this purpose in the linearly extending channel with outlet section \( a = 10 \text{cm}^2 \) it is necessary to set both a constant magnetic field \( B = 10\text{T} \) and a linearly increasing electrical field with the value of coordinate derivative \( E'(x) = 2 \times 10^3 \text{ V/cm} \).

4. In a simulated mode the neon was considered as a working body with stagnation parameters at the inlet of the channel \( P_s = 1\text{MPa}, T_s = 2000\text{K} \) and with the value of mass flow 70g/s. In the established periodic acceleration mode the average increase of gas velocity per 1m of the channel length is about 15 km/s. The used electrical power (without considering the expenses for the initiation of initial temperature perturbations) is equal \( 8 \times 10^6\text{W} \) thus, efficiency of the MHD accelerator is 95% at the thrust force about 1000N.

5. The model of ionization and recombination kinetics has been created for the simulation of the gas-plasma MHD processes in the noble gas flow.

6. The possibility of realization of the MHD process with nonequilibrium ionized plasma clot in the condition of “frozen” ionization, has been shown. This regime is the most effective for transformation of the heat energy into the electric one and, besides, in this condition the plasma clot do not run the risk of the ionization-overheating instability.

7. The results of simulation of the process in the Faraday MHD generator channel has demonstrated the possibility of reaching of the high performance in the both enthalpy extraction ratio and adiabatic efficiency. The effective working regime has been obtained at the stagnation pressure of working medium 2MPa, which allows to use the gas-cooled nuclear reactor as a heat source in the one-contour cycle. In this case the space power plant will be compact and with low specific mass.

References


