10. NUMERICAL INVESTIGATION OF MGD FLOWS IN THE MODELS OF SUPERSONIC INTAKES

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Abstract. The flows of a weakly ionized gas subjected to magnetic field are investigated as applied to shock tube experiments conducted in the Ioffe Institute. Numerical simulation is carried out within the MGD approach using the simplest physical and gas dynamic models. Numerical solutions are obtained with the explicit high-resolution Godunov-type computational schemes.

1. Introduction

The idea to control electrically conducting flows with applied magnetic field is successfully exploited in a number of technical applications. The paper considers the prospects of this mode of flow control as applied to operation of the intakes of supersonic aircraft. The method to be discussed consists in preliminary ionization of the incoming gas and subsequent control over the ionized flow by applied magnetic field. To estimate the effects of magneto-gas dynamic interaction on plasma flow deceleration, numerical simulation of internal supersonic electrically conducting gas flows subjected to applied magnetic field is performed using the simplest physical and gas dynamic models. The calculations are carried out over the range of conditions to be investigated in the experiments using Small and Big Shock Tubes of Ioffe Institute. The results show qualitative agreement with the experimental data.

2. Statement of the Problem

The flows of electrically conducting gas subjected to magnetic field are governed by gas dynamic equations complemented with the MGD terms

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0
\]

\[
\frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = -\nabla \cdot \mathbf{P} + \mathbf{F}
\]

\[
\frac{\partial (p \mathbf{V})}{\partial t} + \nabla \cdot (p \mathbf{V} \mathbf{V}) = -\nabla \cdot q - P : \nabla \mathbf{V} + Q
\]

Maxwell equations

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu (\mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}) ,
\]

\[
\nabla \cdot \mathbf{D} = \rho_e , \quad \nabla \cdot \mathbf{B} = 0.
\]

and generalized Ohm’s law

\[
\mathbf{j} + \rho_e (\mathbf{j} \times \mathbf{B}) = \rho_e \mathbf{V} + \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B}),
\]

Here \( \rho, \mathbf{V}, e \) are the density, the velocity and the specific internal energy of a gas; \( P \) is the stress tensor, \( q \) is the heat flux density, \( \mathbf{F} \) is the ponderomotive force, \( Q \) is the Joule heat release, \( \mathbf{B} \) is the magnetic field induction, \( \mathbf{E} \) is the electric field strength, \( \mathbf{D} = \epsilon \mathbf{E} \) is the electric field induction, \( \mathbf{j} \) is the electric current density, \( \rho_e \) is the charge density, \( \epsilon \) is the permittivity, \( \mu \) is the permeability, \( \sigma \) is the electric conductivity, \( \mu_e \) is the electron mobility. Generalized Ohm’s law (2.3) is written neglecting the ion slip and the electron pressure gradient.

Ponderomotive force and Joule heat release are given by formulae:

\[
\mathbf{F} = \rho_e \mathbf{E} + \mathbf{j} \times \mathbf{B}, \quad Q = \mathbf{j} \cdot (\mathbf{E} + \mathbf{V} \times \mathbf{B})
\]

The flows under study meet certainly the MGD approach conditions:

\[
V_0 << c, \quad E_0 \leq V_0 B_0, \quad \tau_g = L/V_0 > \omega_p^{-1}
\]

where \( L \) is the flow length scale, \( c \) is the light speed, \( \omega_p \) is the plasma frequency, and subscript 0 denotes the scales of gas velocity, electric field strength and induction of magnetic field.

MGD approach allows neglecting the displacement current \( \frac{\partial \mathbf{D}}{\partial t} \), the convective current \( \rho_e \mathbf{V} \), and \( \rho_e \mathbf{E} \) contribution to the ponderomotive force [1]. Beside that, the analysis uses the following assumptions:

(a) \( \alpha << 1 \),
(b) \( \text{Re} = V_0 L/\eta_0 >> 1 \),
(c) \( \text{Re}_m = V_0 L \alpha \mu << 1 \),

where \( \alpha \) is the ionization degree, \( \text{Re} \) is the flow Reynolds number, \( \text{Re}_m \) is the magnetic Reynolds number, \( \eta_0 \) is the scale for gas kinematic viscosity.
Assuming (a), one can neglect ionization effect on thermodynamic properties and regard plasma as a perfect gas with constant heat capacity. Assumption (b) allows the use of an inviscid gas model provided that flow separation does not occur. Assumption (c) enables one to neglect the induced magnetic field, which simplifies the problem reducing the number of dependent variables governed by the Maxwell equations.

Ignoring the induced magnetic field, one can take magnetic induction to be that of applied magnetic field. The latter being known, the ponderomotive force and Joule heat release can be evaluated provided that the electric current density and the electric field strength are available. The simplest way to determine these quantities is to use the generalized Ohm's law (2.3) in which the electric field strength is found through the external load coefficient $k$:

$$\vec{E} = -k(V \times \vec{B}) \quad (2.10)$$

More accurate way is to determine the electric field strength from Maxwell equations (2.2). In the case of a stationary magnetic field, the Faraday law (the first of equation (2.2)) reads:

$$\nabla \times \vec{E} = 0,$$

which implies

$$\frac{\partial}{\partial t} \vec{E} = -\nabla \varphi \quad (2.11)$$

where $\varphi$ is the electric field potential. Generalized Ohm's law takes the form:

$$\vec{j} = \sigma(-\nabla \varphi + \vec{V} \times \vec{B}) \quad (2.12)$$

The electric field potential is found from the condition $\nabla \cdot \vec{j} = 0$ that follows from the second of Maxwell equations (2.2). Along with (2.12) this yields Poisson equation

$$\nabla \cdot [\sigma(-\nabla \varphi + \vec{V} \times \vec{B})] = 0 \quad (2.13)$$

Wall boundary conditions for equation (2.13) are given by the fixed values of the electric field potential on the electrodes and by zero value of the electric current density normal to the insulator walls. The inflow and outflow boundary conditions depend on the duct geometry and the electrode system design.

With the above assumptions, the MGD flows under study are governed by the system of Euler equations with additional terms taking into account the MGI. This system is a hyperbolic one for non-stationary and stationary supersonic flows and possesses elliptic properties for stationary flows with subsonic zones. The initial and boundary conditions for gas dynamic functions are formulated using the characteristic theory [2].

Computational procedures for solving gas dynamics equations are based on the explicit high-resolution Godunov-type scheme suggested by Rodionov [3] for space-marching calculations of stationary supersonic flows. The elliptic properties inhering the governing equations in the case of flows with subsonic zones prohibit the use of space-marching calculation procedure. This being the case, the stationary solutions are obtained with the time-asymptotic technique. The time-marching procedure is similar to that advancing solution in the hyperbolic $x$-coordinate for a stationary supersonic flow. The above methods allow shock-capturing calculations ensuring monotonicity and second-order accuracy in the regions with smooth function variation.

3. Results of Calculations

The results to be discussed pertain to plane and three-dimensional MGD flows in the intake models used in the shock tube experiments.

Fig.1. MGD section of the SST setup.

Fig.1 presents an outline of the MGD section of the experimental setup using Small Shock Tube of the Ioffe Institute. Figures 1-7 denote the shock tube, the vacuum seal, the diaphragm, the vacuum chamber wall, the nozzle, the diffuser and the electrodes, respectively. The magnetic field is applied normal to the figure plane. So the flow under study is two-dimensional one. At the inflow and outflow boundaries, partial derivatives of the electric field potential with respect to the longitudinal coordinate were assumed to be equal to zero, which accounts for rapid diminution of the electric field with a distance from the electrode zone.
Figs. 2 through 5 present some results of numerical simulation of MGD flows in the above setup. In these calculations the Hall parameter, $\beta$ was assumed to be equal to zero, the electrode voltages, $\Delta \phi$ were assumed to be $20V_0BL$.

Figs. 2 present Mach number contours in the MGD section without the magnetic field (a), at the Stuart number $S=0.00005$ (b), and $S=0.0005$ (c). Here, the Stuart number is defined with the length scale $L=1\text{mm}$ which is less, approximately, by two orders of magnitude than the length of the MGI zone. In the case (c), the MGI decelerates plasma flow even within the supersonic nozzle. With increasing MGI, the shock intersection point shifts upstream. In all above cases, the flow remains to be steady and supersonic one.

Figs. 3 present the total pressure loss and static pressure profiles in the outlet cross section of the MGD flows shown in Fig. 2. It is seen that along with an increase of the static pressure the MGI leads to substantial loss of the total pressure. The latter is explained, obviously, by the Joule heat release.

The next two figures present a comparison of computations with the experimental data. Fig. 4 depicts a position of the shock intersection point versus the magnetic field induction. Solid line presents computations; the circles stand for the experimental data.

Fig. 2. Mach number contours in the SST setup. (a) $S=0.0$, (b) $S=0.00005$, (c) $S=0.0005$, $\Delta \phi=30V$

Fig. 3. Outlet total pressure loss and static pressure profiles

Fig. 4. Dependence of the shock intersection position on the induction of applied magnetic field.
Figs. 5 show plasma radiation measurements in the diffuser model (top) and the computed temperature field (bottom).

The next figure presents calculations of MGD flows in the experimental setup using Big Shock Tube of the Ioffe Institute. In this case, the magnetic field is aligned with the y-axis and the electrode system is that of Faraday MGD generator with continuous electrodes. Taking into account the Hall effect, the flows under study are three-dimensional. With sufficiently large Stuart number, the flow becomes non-stationary one. Figs. 6 (left and right) present Mach number contours in the horizontal (top) and vertical (bottom) diffuser symmetry planes for two consecutive moments following one after another through 10μs. It is seen a normal shock wave and a subsonic zone behind it which move upstream.

Fig. 6. Unsteady three-dimensional flow in the BST setup. \( S=0.005, \beta=1.0, k=0.1 \).

4. Conclusions

Numerical investigation of two- and three-dimensional flows in the MGD sections of two experimental setups has been performed. Numerical simulation indicates a possibility for plasma flow deceleration down to subsonic speed and the onset of non-stationary flow regime with increasing the MGI. The results which are in a qualitative agreement with the shock tube experiments prove that plasma flow deceleration by applied magnetic field is accompanied with a substantial loss of the total pressure.

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References