Abstract. Inviscid flow around a cone-cylinder supersonic body with forebody filamentary streamer-discharge structure is analyzed numerically. Two limiting cases are considered for possible streamer-channel development corresponding respectively to the gas pressure $p$ and gas density $\rho$ constant during this process. For a freestream Mach number $M=2$, the computations predict around 20-30% drag reduction and 0.3-0.4 drag reduction energetic efficiency. Drag reduction energetic efficiency weakly depends on the relative inter-filamentary distance and dramatically increases with increase of the freestream Mach number. For a $p=$const model it becomes greater than unity for $M>3$ and reaches 3.7 for $M=6$. The high efficiency of the drag reduction for $M>3$ is applicable to both small and realistic sized vehicles.

1. Introduction

The idea of the shock wave (SW) diffusion and attenuation due to interaction filamentary discharge structure was checked and analyzed numerically in [1,2] for 1-D SW propagating through an impulse discharge cold Air plasma. The results obtained were encouraging in depicting existing experimental data trends.

Analysis of the experimental data in the open literature strongly suggests multi-filament structure creation during the gas breakdown process both for direct current impulse [3] and high frequency corona discharges [4], when the gas pressure is greater than few hundred Torr. Thus the application of a filamentary discharge to attenuate the bow shock in front of the supersonic body looks attractive for atmospheric conditions.

Self-consistent numerical modeling for multi-streamer discharge development and its interaction with the supersonic flow is a complex problem. As part of its solution, it is necessary to assess the performance benefits and the energetic practicality of the attendant streamer generation parameters.

This paper discusses a 2-D axisymmetric numerical analysis of the bow SW modification and drag reduction of a supersonic body with forebody filamentary discharge-induced temperature inhomogenities. Their attendant drag reduction is analyzed depending on freestream Mach number and filamentary structure geometry.

2. Assessment of the streamer discharge filament parameters.

Streamer breakdown for a pulsed uni-polar electric potential in a single electrode system (the second electrode is at infinity) assumes creation of a high conductivity channel. The breakdown seems to be completed when additional ionization is caused either by association process for excited molecules, or by associative ionization $N+O\rightarrow NO^++e$ [5,6]. If additional ionization is not maintained by both of these mechanisms, the streamer channel decays due to recombinaction in around 10-100ns, because the electron concentration at the head of the streamer is around $10^{14}-10^{15}$cm$^{-3}$. The associative ionization becomes essential when the temperature inside streamer channel is greater than 4500-5000K. It can be realized either by fast repetition of decaying short streamers, or by gas heating in an initially long streamer channel without complete decay of the channel conductivity. These scenarios depend on initial absolute value and slope of the electrode potential.

Experimental investigations of HF streamer discharge in Air are summarized in [7]. For spike electrode HF voltages up to 13kV with frequencies $f=$3.3 to 10MHz the discharge development exhibits streamer creation for positive electrode polarity and disappearance when the electrode polarity changes the sign. Streamer repetition leads to formation of a luminous discharge stem, which does not disappear during the oscillations of the external electric field. It moves forward with velocity around 103cm/s much less than the velocity of streamer propagation ($10^{8}-10^{9}$cm/s). The higher HF voltage and $f$ value, the bigger the stem velocity.

Starting from the fact that filaments exist in an HF discharge in front of the supersonic body [4] and accounting for the streamer breakdown process [5,6], it seems possible to assume the gas temperature $T_{a}$ inside the filament to be approximately 5000K. For this case, the electron density is of the order of $10^{12}-10^{14}$cm$^{-3}$, and its mean energy is $T_{e}e\approx1-3$eV. The effective plasma temperature for a multi-component gas mixture representing plasma $T_{e}=T_{ch}+\alpha T_{a}$ is close to the neutral molecule temperature $T_{a}$ for aforementioned
parameters, where $\alpha_i$ is the ionization degree of plasma.

3. Statement of problem

To investigate plasma-aerodynamics augmentation for filamentary structure forward of a supersonic body, a cone-cylinder body is chosen as an example. Concentric cylindrical layers model the filamentary structure. The distance between the layers is $d=1-2\text{mm}$ and the layer width is $d_s=0.25-0.5\text{mm}$. The filament length $l_f$ is varied from $1\text{cm}$ to $3\text{cm}$. The cylinder radius $R=1\text{cm}$ and the cone half-angle is $35.5^\circ$. The freestream and filament parameters are $M_f=2-6$, $p_v=0.32\times10^5\text{Pa}$, $\rho_v=0.613\text{kg/m}^3$ and $T_f=5000\text{K}$.

For the problem of streamer discharges in front of a supersonic bodies, the characteristic gasdynamic time $\tau_g$ is around $l_f/V_v\sim100\text{mcs}$, where $V_v$ is the freestream velocity. Depending on the streamer stem (or filament) parameter evolution/transient time $\tau_f$, two limits for the statement of problem are possible. For $\tau_f\ll\tau_g$, we can use the filament parameters as initial conditions for gasdynamic equations. If $\tau_f\geq\tau_g$, the simultaneous/coupled solution of the filament development and gas flow variation is essential, and the problem becomes much more complicated.

If $\tau_f\geq\tau_g$, the discharge arises in a “clinging” mode, because the filaments are convected by the oncoming flow during their formation. For practical applications, we are interested in the opposite case corresponding to $\tau_f\ll\tau_g$, where discharge filaments pierce through the bow shock. Only this mode can diffuse and attenuate the bow SW, and it is considered in the current paper.

The initial inviscid flow pattern with a bow shock in front of the supersonic cone-cylinder is computed without filaments using Godunov’s first order algorithm. For the time instant we denote as $t=0$, the discharge is turned on, and high temperature filaments are created instantaneously. No energy flux into the filaments is assumed after their formation. Accordingly, the initial conditions for the space distribution of the gas parameters relevant for $t=0$ are that in the space between the filaments the flow parameters remain undisturbed, whereas inside the filaments the gas velocity remains undisturbed but gas temperature, density and pressure correspond to filamentary conditions.

There are two opposite limits for the filamentary evolution process. The first is constant pressure, ($p=\text{const}$) during filament formation and corresponds to the time of sound propagation across the filament $\tau$, being much less than the filament channel formation time $\tau_f$ ($\tau_\gamma<<\tau_f<<\tau_g$), the reverse limit ($\tau_f<<\tau_g<<\tau_g$) corresponds to constant density ($p=\text{const}$).

4. Results and discussion.

The calculated Mach number contours at different times are shown in Fig.1 for the $p=\text{const}$ model and $M_f=2$, $l_f=1\text{cm}$, $d=1\text{mm}$, $d_s=0.5\text{mm}$. Red and blue colors correspond respectively to $M<1$ and $M>1$ values. The arrow indicates the boundary of the computational domain. The drag coefficient $c_x$ time variation is presented in Fig.2 for different $l_f$ values. Solid curves correspond to the $p=\text{const}$ model for filament formation. The dashed curve corresponds to $p=\text{const}$. If $p=\text{const}$ during the filament channel formation, an approximately 20% drag reduction can be achieved.

Consider the energetic efficiency of drag reduction $\eta_E$ defined by the ratio of the drag-change power to the electrical power $Q_f$ needed to create the relevant filamentary structure

$$
\eta_E = \frac{\rho_s V^2 \mathcal{R}^2 \int_0^{\tau_f} (c_{x0} - c_x) \, dt}{2Q_f}.
$$

If all the energy $Q_f$ goes into the gas heating from the initial temperature $T_{x0}$ to the final filament temperature $T_f$ (100% electrical conversion efficiency), the expressions for $\eta_E$ are modified to

\[\text{Fig.1 Mach number contours in the flow around the cone-cylinder for } l_f=1\text{cm}.\]
\[ \eta_E = \frac{\gamma M^3 (1 + d/d_s)}{5 T_f / T_{\infty}} \int_0^t (c_{x0} - c_x) \, dt \]  
for \( p=\text{const} \) model,

\[ \eta_E = \frac{\gamma M^3 (1 + d/d_s)}{5 \ln(T_f / T_{\infty})} \int_0^t (c_{x0} - c_x) \, dt \]  
for \( p=\text{const} \) model,

where \( a_v \) is the freestream speed of sound.

For the \( p=\text{const} \) model, the efficiency is around 30% and weakly depends on the filament length.

Figures 4 and 5 represent the results of the computations for different freestream Mach numbers and different ratio of the inter-filament distance and filament diameter for \( l_s=2R=2\text{cm} \) and \( T_f=5000\text{K} \). The freestream conditions are the same as previously mentioned, i.e., \( p_\infty=0.32\times10^5 \text{Pa}, \rho_\infty=0.613\text{kg/m}^3 \).

The drag reduction efficiency for the \( p=\text{const} \) model weakly depends on \( d/d_s \) (Fig.4) and dramatically increases with increasing Mach number, approaching 370% for \( M_f=6 \) both for \( d/d_s=2 \) and 8 (Fig.5). Thus, the filamentary structure in front of a supersonic body is beneficial for \( M_f>3 \), when \( \eta_E \) becomes greater than unity.

![Fig.2](image_url)  
**Fig.2** Drag coefficient over time variation for \( l_s=1\text{cm} \) (1,1a), 2cm (2), and 3cm (3); solid lines – \( p=\text{const} \) model, dashed line - \( p=\text{const} \) model (\( M_f=2, d=1\text{mm}, d_s=0.5\text{mm} \)).

![Fig.3](image_url)  
**Fig.3** Drag reduction energetic efficiency for \( p=\text{const} \) (1) and \( p=\text{const} \) (2) models for \( M_f=2, d=1\text{mm}, d_s=0.5\text{mm} \).

The calculated \( \eta_E \) as a function of the filament length for \( M_f=2, T_f=5000\text{K}, T_{\infty}=182\text{K}, d=1\text{mm}, \) and \( d_s=0.5\text{mm} \) is shown in Fig.3. For the \( p=\text{const} \) model, the initial pressure jump increases the \( c_x \) value to 7.6 1 compared to \( c_{x0}=1.16 \) followed by a \( c_x \) decrease of 40-50%. The net drag reduction is close to zero ( \( \int_0^t (c_{x0} - c_x) \, dt \to 0 \)).

1 Because of large scale this point is not seen on curve 1a in Fig.2.
is clear. This is because the bow shock intensity and pressure drag increase for increasing hypersonic Mach numbers. This leads to increased bow shock attenuation and pressure drag reduction due to filamentary structures upstream of forebodies.

The fact that $\eta_e$ weakly depends on the $d/d_s$ ratio is encouraging for scaling the results from $R=1\text{cm}$ to $R$ values corresponding to realistic vehicles. Since the streamer channel diameter $d_s$ cannot be increased for long filaments necessary for real size vehicles ($l_s\sim R$), the ratio $d/d_s$ will increase with increasing $R$ because of technical and construction reasons. But according to the results presented in Figs.4 and 5, it can be expected that $\eta_e$ for real vehicles will remain close to those for small ones.

5. Summary

The inviscid flow about a cone-cylinder supersonic body with a forebody filamentary discharge plasma was analyzed numerically. The filament parameters were estimated from experimental data as well as physical models for streamer channel development.

Two limiting cases were considered for streamer-channel development corresponding respectively to the gas pressure and density invariant during the high temperature filament formation.

At $M_s=2$, the computations predict an approximately 20-30% drag reduction for the $p=\text{const}$ model and 40-50% drag decrease for the $U=\text{const}$ model. The time-averaged effect of drag reduction is respectively around 20% and zero for these two models. The energetic efficiency of drag reduction for $M_s=2$ is about 0.3-0.4 for the $p=\text{const}$ case and 0.01 for $p=\text{const}$ model.

Drag reduction energetic efficiency weakly depends on the relative inter-filament distance $d/d_s$ and dramatically increases with increasing freestream Mach number. For the $p=\text{const}$ model, it becomes greater than unity for $M_s>3$ and reaches 3.7 for $M_s=6$.

The drag reduction efficiency weak dependence on the relative inter-filament distance $d/d_s$ suggests high efficiency results for $M_s>3$ can be realized both for small and realistic sized vehicles.

Streamer channel development physics is extremely important resulting in either drag reduction or sharp drag increase. Both outcomes can be realized depending on the applied impulse voltage, its slope, and the shape of the electrode.

Filamentary temperature transients crucially depend on plasma kinetics inside the filament. This feeds the conductivity of the streamer channel and the possibility of energy input from to the streamer current, including Joule heating. Thus, the filamentary thermalization should be analyzed carefully to obtain a self-consistent model for streamer discharge aerodynamic performance benefits.

Experiments for multi-streamer discharges in front of supersonic bodies are necessary to demonstrate the efficiency of multi-filamentary structure creation and resulting aerodynamic augmentation. They are also critical for validation of our emerging predictive models that need to be integrated into systems design.

From the foregoing discussion it is evident that streamer channel formation numerical simulation is an extremely intricate problem. Reliable predictions can be obtained only on the basis of the joint experimental, theoretical, and computational effort.

References