6. ABOUT ACCURACY OF TEMPERATURE MEASUREMENT FOR THIN FIBER OF NON-HOMOGENEOUS NON-EQUILIBRIUM ELECTRIC-DISCHARGE PLASMA

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Abstract. For widely used experimental technique for measurement of plasma translational temperature [1-3], based on analysis of viewed spectrum of molecular nitrogen, calculation estimation has been obtained for upper bound of measurement error for maximum translational temperature for non-homogeneous fiber of non-equilibrium dissociating plasma.

Scope of parameters has been considered, which is characteristic of electro-discharge plasma [3-5]. Probe dependencies on radial coordinate have been used for temperature and concentration. Calculations of spectral dependence of radiation intensity have been carried out for 0-0 transition of second positive system of molecular nitrogen.

Strict conditions for accuracy of measurement of plasma translational temperature have been considered, when dependence of temperature on radial coordinate does not have the only strong maximum, and vibrational temperature of plasma exceeds the translational temperature strongly, so that dissociation of plasma is very significant.

It has been shown for non-equilibrium plasma, that plasma maximum translational temperature can differ from measured effective temperature more than by 500K, when measured temperature of plasma is on the level of 2300K. Such error of measurement is substantially greater, than 50K level, which has been obtained previously for non-dissociating plasma and for temperature dependence on coordinate with the only strong maximum [1-3].

It has been shown also, that for plasma at equilibrium, error of measurement can mount ~15000K, when intensity radiation of very thin hot kernel with temperature 20000K is substantially less, than intensity of radiation of relatively cold 1mm radius envelope.

1. Introduction

A purpose of this work is to find numerical level for maximum error of measurement of temperature for non-homogeneous fiber of non-equilibrium dissociating plasma of nitrogen for a case of widely used experimental technique [1-3], which is based on analysis of viewed molecular spectrum of nitrogen. When this technique is used, maximum translational temperature of plasma is determined by means of comparison of experimental spectrum with a number of spectrums, which have been calculated for different temperatures. The temperature, for which the spectrum calculated is close at most to experimental spectrum, is usually considered to be equal to maximum translational temperature for plasma fiber.

Strictly speaking, technique described above gives a possibility to find not maximum translational temperature, but some effective temperature of plasma, which depends on temperature dependence on coordinate along ray of observation [1,2]. Nevertheless for some important cases effective temperature and maximum temperature are practically equal to each other.

For example, it has been shown at [1,2], that when dissociation can be neglected, and temperature dependence on coordinate has the only strong maximum, then intensity of radiation for hot volumes of plasma with temperature near to maximum overwhelms intensity of radiation emitted by volumes, which are less heated. Overwhelming of contribution to intensity of radiation from volumes with temperature near to maximum temperature is due to exponential dependence of radiation intensity on temperature [1,2].

Maximum temperature of plasma in this case is near to effective temperature, which can be find with accuracy not worse, than 50K, - also due to exponential dependence of radiation intensity on temperature [3]. Correspondingly, the same high is an accuracy of measurement of maximum translational temperature of plasma. Cases, when effective temperature strongly differs from maximum temperature, as it is known by authors, have not been considered previously.

At this work more strict conditions have been considered, as compared with previous works. At first we have considered high temperatures, when dissociation of plasma is strong. It has been supposed also, that dependence of temperature of plasma on coordinate does not have the only strict maximum. Level of measurement error, which has been found for these conditions, can be considered as an estimation for upper bound for error of measurement of maximum temperature.

2. Basic assumptions

Similar to [1-3], it has been supposed, that populations of energetic levels at non-equilibrium plasma can be described by Boltzman relation; translational and rotational temperatures are equal.
to each other, and can strongly differ from vibrational temperature.

Scope of physical parameters has been considered, which is typical of electric-discharge [1-3]: translational temperature of plasma is 300-20000K, vibrational temperature of plasma is 300-10000K, initial pressure is on the level of 10Torr. Electric field is in the range $3 \cdot 10^{-16} < E/N < 6 \cdot 10^{-16}$ Vcm$^{-2}$, concentration of electrons is on the level of $10^{10} - 10^{12}$ cm$^{-3}$. Mean energy of electron is 0.5-3eV. Typical viewed dimension of plasma fiber is 1-2mm.

Calculations of intensity of radiation have been carried out for 0-0 transition of second positive system of molecular nitrogen. Intensity of radiation, which is due to electron-vibrational-rotational transitions, has been calculated on a base of known classic formulae [6,7], tabulated data [8], and under assumption, that plasma is transparent. Width of apparatus function has been assumed to be equal to 1-1.2 angstroms, similar to [3].

Probe dependencies of temperature and concentration on radial coordinate have been used. Theoretical modeling of non-homogeneous, non-equilibrium plasma based on numerical solution of system of kinetic equations for populations of energetic levels, together with system of equations of gas dynamics is not in the scope of this investigation.

At this work two-steps dependence of temperature on radial coordinate has been considered. Such dependence does not have obligatory the only strong maximum, and can be used as an example to obtain upper estimation for error of experimental technique under consideration. It is due to at this case intensity of radiation from hot internal kernel of fiber can be of the same order of magnitude, or even less, than intensity of radiation of relatively cold envelope, which can have substantially greater value of volume, as compared with the kernel.

At the case of equilibrium plasma with temperature, which is not greater, than 3000K, dissociation can be neglected indeed [9], as it has been done at previous works [1-3]. But on a practice plasma can be substantially non-equilibrium one, because main part of energy, which is passed into plasma, transforms to excitation of vibrational levels, and it gives rise to a situation, when vibrational temperature can exceed translational temperature by some times, or even by the order of magnitude [4,5].

It is known, that for pressure greater, than 1.5 Torr, for conditions of electric-discharge plasma, dissociation of molecular nitrogen is near to entirely due to excitation of vibrational levels of basic electron state [5]. At this work calculations have been carried out using [10], which have shown, that for vibrational temperature near to 10000K, and for translational temperature 3000K, dissociation of molecular nitrogen can be the same strong, as equilibrium dissociation for temperature 10000K. So that, even when translation temperature is substantially less, than temperature of equilibrium dissociation, for non-equilibrium electric-discharge plasma dissociation should be taken into account obligatory.

To estimate numerical value of concentration of molecules of dissociating plasma, we have used an expression for equilibrium constant [9].

For the case of non equilibrium plasma we have used the same expression, as for equilibrium plasma, where temperature has been supposed to be equal to vibrational temperature of plasma.

This approach seems to be adequate, when a time of VV-relaxation at discharge is substantially less, than a time of dissociation, and, in turn, a time of dissociation is substantially less, than a time of VT-relaxation. For some ratio of physical parameters at electric-discharge such relation between characteristic times can take place. Indeed, in accordance with [4] (Table 11.3), for electric field $E/N = 3 \cdot 10^{-16}$ Wt cm$^{-2}$, for concentration of electrons $n_e = 10^{11}$ cm$^{-3}$, for $T_g = 500$K, for initial pressure of molecular nitrogen 5Torr we have: time of VV-exchange $5.5 \cdot 10^{-5}$s, time of eV-exchange $2 \cdot 10^{-3}$s, time of dissociation $1.6 \cdot 10^{-1}$s, time of dissociation due to direct electron collision $0.5$s, time of VT-relaxation $2.5 \cdot 10^{-2}$ s. For electric field $E/N = 6 \cdot 10^{-16}$ Wt cm$^{-2}$, and for the same concentration of electrons, temperature of gas and initial pressure of molecular nitrogen we have: time of VV-exchange $5.5 \cdot 10^{-5}$s, time of eV-exchange $10^{-6}$s, time of dissociation $1.6 \cdot 10^{-1}$s, time of dissociation due to direct electron collision $0.5$s, time of VT-relaxation $2.5 \cdot 10^{-2}$s.

Calculation of intensity of radiation

As an example, we consider 0-0 transition of band $C^3Π_L - B^3Π_g$ of molecular nitrogen (second positive system). For transparent plasma spectral intensity of radiation is defined by following relation:

$$I_\lambda = \frac{R_{\text{view}}}{\int_0 \sum_{\text{lines}} j_{f}(\lambda) I(\lambda) d\lambda}$$

Here $j_{f}(\lambda)$ - radiation coefficient, $I(\lambda)$ - contour of line ($\int_{\lambda} I(\lambda) d\lambda = 1$), $r=0$ corresponds to
the axe of plasma fiber, $R_{viewer}$ - distance from the axe of fiber to registering apparatus.

Radiation coefficient for electron-vibrational-rotational (eVR) line of band can be calculated using following known formula [6]:

$$\gamma_{j,j'} = \frac{16\pi^2 c}{3} \omega N \frac{g_e g_n}{Q_n} S_{j,j'} q_{j,j'} \frac{S_e}{g_e} \times \exp\left[\frac{-\hbar c}{kT} (E_e' + E_v' + E_J')\right]$$

Here $c$ - speed of light, $\omega$ - frequency of radiation, cm$^{-1}$, $N$ - number of molecules inside radiating volume, $g_e'$, $g_n$ - electron and nuclear statistical weights, $Q_j$, $Q_n$ - whole and nuclear statistical sums, $S_{j,j'}$ - Henle-London factor, $q_{j,j'}$ - Frank-Kondon factor, $S_e$ - strength of electron transition, $E_e'$, $E_v'$, $E_J'$ - energy of upper electron level, energy of upper vibrational level, energy of upper rotational level.

At the case of non-equilibrium plasma, rotational temperature should be put in formula for intensity of radiation, and here the rotational temperature is practically equal to translational temperature during all time of discharge.

$^3\Pi$ state of nitrogen molecule refers to a case of a-bond in terms of Gund [6], when $J=\Omega, \Omega+1, \ldots$, where $\Omega$ - sum of projection of whole spin on axe between nuclei and projection of orbital moment of electrons on the same axe. Following numerical data have been used [7]:

$$E_e' = 89105 \text{ cm}^{-1}, \quad E_v' = 59583.4 \text{ cm}^{-1}$$

$$E_J' = \hbar c \omega_n \left( \nu + \frac{1}{2} \right)$$

$$E_J' = B_e J(J+1), \quad \omega_n' = 2035 \text{ cm}^{-1}, \quad \omega_n = 1734 \text{ cm}^{-1},$$

$$B_e = 1.826 \text{ cm}^{-1}, \quad B_e = 1.638 \text{ cm}^{-1}$$

For Frank-Kondon factor of 0-0 vibration we have used value 0.455, which has been tabulated at [8].

For both energetic states $^3\Pi_u$ and $^3\Pi_g$ projection of whole orbital moment of electrons on an axe of molecule is characterized by quantum number $\Lambda=1$, so that $\Delta\Lambda=0$. In this case for Frank-Kondon factors we have [6]:

$$S_{j,j'}^{J+1} = S_j^J \frac{(J+1+\Lambda)(J+1-\Lambda)}{J+1},$$

$$S_{j,j'}^{J-1} = S_j^J \frac{(J^2-\Lambda^2)}{J},$$

$$S_j^J = S_j^J \frac{(2J+1)\Lambda^2}{J(J+1)}$$

Here $J$ - rotation quantum number for down energetic level.

Coefficient of radiation for spectral line represents integral of spectral intensity of radiation over spectrum. Estimations of width of eVR-lines show, that characteristic width of lines is of the order of 0.01 angstrom. Indeed, for doppler broadening we have [6]:

$$\gamma_D = 4.3 \times 10^7 \left( \frac{\lambda}{\mu} \right)^{1/2}$$

$$= (\lambda=3300 \text{ Å}, T=3000 \text{ K}, \mu=28) = 0.015 \text{ Å}$$

Collisional resonance broadening can be estimated using formula [6]:

$$\gamma = \frac{\gamma}{m \omega} \left( \frac{g_f}{g_j} \right)^{1/2}$$

$$= (\omega=2.8 \times 10^{10} \text{ s}^{-1}, m=91 \times 10^{28} \text{ g}, f=0.09, \omega=3 \times 10^4 \text{ cm}^{-1}, \nu=9 \times 10^2 \text{ s}^{-1}) = 10^{-7} \text{ s}^{-1}$$

Characteristic width of apparatus function for experimental equipment [3] is of the order of 1 Å, and hence it is substantially exceeds the width of spectral lines. Thus, apparatus, which is used at [3], resolves spectral lines partially.

At Fig.1,2 spectral dependencies of radiation intensity are shown, which have been calculated for different translational temperatures. Comparison of calculated spectra with experimental spectra, which are also shown at the same figures, gives the possibility to find effective temperature, which is usually considered, as maximum translational temperature at plasma fiber [1-3].

![Fig.1.](image-url) Spectral dependencies of intensity calculated for $T=500, 550, 600 \text{ K}$ as compared with experimental data (black).
Effective temperature can be determined using such technique with an accuracy of the order of 50K due to strong, exponential dependence of radiation intensity on temperature. At the same time assumption, that effective temperature is very close to the maximum temperature, can be wrong for some cases. At this work two examples will be demonstrated, when effective temperature differs from maximum temperature substantially greater, than by 50K.

4. Calculation of dissociation rate of nitrogen

Dependence of relative concentration of nitrogen molecules on temperature, as it has been described at 2 Basic assumptions, are shown at Fig.3. For temperature dependence of equilibrium constant empiric relation [9] has been used. It can be seen, that for temperature 10000K concentration of nitrogen molecules is by 4 orders of magnitude less, as compared with concentration for temperatures, which are less, than 3000K.

When translational temperature of electric-discharge plasma does not exceed a level of 3000K, as at [3-5], dissociation of equilibrium nitrogen plasma can be neglected (Fig.3).

Plasma of electric-discharge can be non-equilibrium one, so that vibrational temperature of plasma strongly exceeds translational temperature. It is due to the fact, that a main part of energy of electromagnetic field is passed to excitation of vibrational energetic levels of molecules, and a time of vibration-translation exchange is relatively large [4,5]. As it has been pointed out at [5], for pressures more than 1,5 Torr and for conditions, which are typical of electric-discharge, dissociation of N2 is only due to excitation of vibrational levels of basic electron state. At this work calculations have been carried out to find rate of equilibrium dissociation and to find non-equilibrium coefficient correction factor $Z(T, T_v)$ using formulae [10], which have shown, that for vibrational temperature of the order of 10000K, and for translation temperature 3000K, dissociation of nitrogen can be of the same significance, as compared with equilibrium dissociation for 10000K. Results of calculations are shown at Fig.4.

Thus, even when translational temperature is significantly less, than the temperature of equilibrium dissociation, it is necessary to take into account dissociation for non-equilibrium plasma of electric-discharge.

An extent of dissociation of nitrogen for known constants of rates of dissociation and recombination can be found by means of solving system of kinetic equations, but approximate solution can be found using more simple algorithm. To find approximate solution, it is sufficient to take into account relation of characteristic times of processes of VV- and eV-exchange, time of dissociation and time of VT-relaxation.

In accordance with [4] (Table 11.3), for electric field $E/N=3\times10^{-16}$ Wt cm$^2$, for concentration of electrons $n_e=10^{11}$ cm$^3$, for $T_{gas}=500$K, for initial pressure of molecular nitrogen 5Torr we have: time of VV-exchange $5.5\times10^{-3}$s, time of eV-exchange $2\times10^{-3}$s, time of dissociation $1.6\times10^{1}$s, time of dissociation due to direct electron collision 0.5s, time of VT-relaxation $2.5\times10^{2}$ s. For electric field $E/N=6\times10^{-16}$ Wt cm$^2$, and for the same concentration of electrons, temperature of gas and
initial pressure of molecular nitrogen we have: time of VV-exchange 5.5×10⁻⁴s, time of eV-exchange 10⁻³s, time of dissociation 1.6×10⁻¹s, time of dissociation due to direct electron collision 0.5s, time of VT-relaxation 2.5×10⁻¹s.

Thus, a time of VV-relaxation at electric-discharge is substantially less, than a time of dissociation, and in turn a time of dissociation is substantially less, than a time of VT-relaxation. Hence to estimate the extent of dissociation of nitrogen at discharge it is proposed to use an expression for equilibrium rate of dissociation, in that in place of translational temperature vibrational temperature should be passed.

5. Examples, when error of temperature measurement is large

Previously, as it has been done at [1-3], by the estimation of the level of measurement error for maximum translational temperature of plasma, a situation has been considered, when intensity of radiation for “hot” volumes of plasma with a temperature, which is close to the maximum, practically overwhelms intensity of radiation from “cold” volumes of plasma with temperature, which considerably less, than maximum temperature of plasma.

Such situation is realized, when a number of radiating molecules with a temperature, which is near to the maximum temperature, is not considerably less, than a number of relatively cold molecules, as compared with difference of radiation intensities per one molecule:

\[ N_{\text{hot}} / N_{\text{cold}} \gg \exp(-E \left( \frac{1}{T_{\text{cold}}} - \frac{1}{T_{\text{hot}}} \right)) \]

Here \( E \) – quantum of radiative transition, \( T \) – translational temperature.

At this case spectral intensity of radiation, which is registered by apparatus, being a value, integrated along a ray of observation, nevertheless with a good accuracy equals to intensity of radiation from only “hot” volumes of plasma, which temperature is near to the maximum one. Registering apparatus practically does not “see” “cold” volumes of plasma.

A situation, which is different in the main, as compared with the situation described above, can be realized, when intensity of radiation from “cold” volumes is not less, than the intensity of radiation from “hot” volumes. For this, a number of radiating molecules inside “cold” volumes should be much larger, than a number of molecules inside “hot” volumes:

\[ N_{\text{hot}} / N_{\text{cold}} \ll \exp(-E \left( \frac{1}{T_{\text{cold}}} - \frac{1}{T_{\text{hot}}} \right)) \]

Temperature dependence of intensity of radiation for molecular nitrogen, integrated over spectrum range 3320-3380 angstroms, is shown at Fig.5. Curve 1 corresponds to the dissociating nitrogen, and curve 2 corresponds to the case, when dissociation has not been taken into account by calculations.

![Graph](image_url)
This dependence is proposed to be used to obtain approximate estimation for \( N_{\text{hot}} / N_{\text{cold}} \). For example, it can be seen at Fig.5, that intensity of radiation for 2500K is approximately by 4 orders of magnitude less, than intensity of radiation for 3000K. It means, that, when a number of “cold” emitters with temperature 2500K is by 4 orders of magnitude larger, than a number of “hot” emitters with temperature 3000K, intensity of radiation from “cold” emitters is approximately equal to intensity of radiation from “hot” emitters, and apparatus registers not only intensity from “hot” emitters, but some substantially different intensity, which can be obtained by means of integration along the ray of observation.

For a case of two-steps dependence of temperature on radial coordinate, a number of “cold” emitters can be significantly larger, than a number of “hot” emitters due to the following two circumstances:

- value of volume of “cold” plasma is considerably larger, than the value of “hot” plasma volume; concentration of “cold” plasma and concentration of “hot” plasma are of the same order of value;
- the main part of molecules of “hot” plasma have been dissociated.

If we take into account all circumstances, which have been pointed out above, we can demonstrate examples, when measurement error for maximum translational temperature is large.

**First example**

At Fig.6 spectral dependencies of intensity of radiation are shown for following three cases:

1— homogeneous plasma fiber with radius 1mm, translational and vibrational temperatures do not depend on radial coordinate and \( T=T_{\text{vibr}} =3000K \). At this case we can assume, that plasma does not dissociate; in addition, we have the only strong maximum for coordinate dependence of temperature.

2— non-homogeneous plasma fiber with radius 1mm, having hot kernel with radius 0.1mm, translational temperature of the kernel is: \( T_{\text{kernel}}=3000K \), vibrational temperature of the kernel is much larger, than the translational one: \( T_{\text{vibr,kernel}}=10000K \); translational and vibrational temperatures of envelope for \( 0.1mm < r < 1mm \) are equal to each other: \( T_{\text{envelope}}=2400K \); 

3— the same, as for 1, except following: \( T=T_{\text{vibr}} =2300K \).

As it can be seen at Fig.6, spectrum of intensity of radiation for case2 is very close for spectrum, obtained for case 3. This takes place, because at the case 2 volume of “cold” envelope is by 2 orders of value larger, than the volume of “hot” kernel, and concentration of molecules in the kernel, which have not dissociated for \( T_{\text{vibr,center}}=10000K \), is by 4 orders of magnitude less, than it takes place for cold envelope (Fig.3). Thus, as it can be seen at Fig.5, larger intensity per molecule for translational temperature 3000K is compensated by a large number of molecular emitters, which have translational temperature 2300K.

Interpretation of spectrum shown at Fig.6 in the way, that spectrum 2 corresponds to the maximum translational temperature of plasma 2300-2400K, would give rise to the error 600-700K by the determining maximum translational temperature of plasma, because in this case, as it has been shown above, maximum translational temperature of plasma is 3000K.

**Second example**

Temperature, that is measured, can be substantially less, than maximum temperature inside plasma fiber not only for the case of non-equilibrium plasma.

To demonstrate this, we have considered plasma at equilibrium. We have calculated intensity for non-homogeneous plasma fiber with hot kernel, and relatively cold envelope. Radius of kernel is 0.05mm, radius of envelope is 1mm. Temperature of kernel is 20000K, temperature of envelope is 4000K. We have compared results for calculation of intensity, that have been obtained for non-homogeneous plasma fiber, with similar results for homogeneous fiber with radius 1mm for different temperatures: 2000K, 2100K, 4000K, 5000K. The results of such comparison are shown at Fig.7.
It can be seen, that neglecting plasma non-homogeneity can result to measurement error of the order of 15000K for maximum plasma temperature. Indeed, spectrum dependence of intensity for non-homogeneous plasma fiber with 20000K for hot kernel is between spectrum dependences for 4000K and 5000K homogeneous plasma fibers.

Conclusions

For widely used experimental technique for measurement of plasma translational temperature [1-3], based on analysis of viewed spectrum of molecular nitrogen, calculation estimation has been obtained for upper bound of measurement error for maximum translational temperature for non-homogeneous fiber of non-equilibrium dissociating plasma.

Scope of parameters has been considered, that is characteristic of electro-discharge plasma [3-5]. Probe dependencies on radial coordinate have been used for temperature and concentration. Calculations of spectral dependence of radiation intensity have been carried out for 0-0 transition of second positive system of molecular nitrogen.

Strict conditions for accuracy of measurement of plasma translational temperature have been considered:
- dissociation of plasma is very significant;
- dependence of temperature on radial coordinate does not have the only strong maximum;
- -vibrational temperature of plasma can exceed the translational temperature strongly.

It has been shown, that:
1. At the case of equilibrium and non dissociating plasma, when vibrational and translational temperatures are near to each other, the accuracy of temperature measurement is not worse, than 50K. Such result is in accordance with results of previous investigations of other authors.
2. Dissociation of non-equilibrium plasma, that is due to high vibrational temperature, when translational temperature is low, can result to considerable decrease of accuracy of temperature measurement. An example of non-equilibrium nitrogen plasma has been demonstrated, when error of measurement is greater, than 500K.
3. For plasma at equilibrium, error of measurement can mount ~15000K, when intensity radiation of very thin hot kernel with temperature 20000K is substantially less, than intensity of radiation of relatively cold 1 mm radius envelope.
4. Experimental technique under consideration, which is widely used to measure translational temperature of non-equilibrium plasma, is recommended to be amplified, to take into account dissociation and non-homogeneity.

References