63. SHORT PULSE PROPAGATION IN DISSIPATIVE AND ACTIVE MEDIA WITH RESONANT RELAXATION

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Abstract. The new analytical representation of fundamental solution (Green's function) describing the short pulse propagation in medium with single process of resonant relaxation is presented. This analytical solution is based on the generalized local response function of linear media. It contains well-known Lorentz's and Debye's models of relaxing media, like particular cases. The changing of pulse shape at propagation, described by obtained solution, shows a variety of forms of pulse propagation and general lows of pulses dynamics beginning from pure relaxation behavior and up to resonant one. The derived solutions are correct and for active media for linear regime of the pulse propagation.

The analysis of dynamics of small perturbations, propagating in non-equilibrium media, is presently the subject of many theoretical and experimental investigations. The interest to this problem is dealt with wide field of applications concerning sound propagation in gases and plasma. For description of non-equilibrium media behavior the model with negative second viscosity is often used.

In the paper the state equation, describing local response of arbitrary linear medium in vicinity of thermodynamic state, is derived by use of thermodynamic approach. This state equation at corresponding values of parameters describes as a particular cases an exponential relaxation processes in medium (Debye's model), as well as resonant relaxations (Lorentz' model):

\[ \sigma - \rho \Delta - \varepsilon = \sigma \int_{-\infty}^{\infty} t e^{-t/\tau} dt \]

where \( \sigma \) - stress, \( \varepsilon \) - strain, \( \rho \) - density, \( \Delta, \tau, \varepsilon, \rho \) - high and low frequency limits of phase velocity in medium, and the basic parameters, which determines attenuation-dispersive properties of medium, are \( \tau \) - relaxation time, \( \Omega \) - resonant frequency, and \( \phi \) - inertial phase. Besides, the following notations are introduced for normalizing multiplier:

\[ \rho = \frac{\Omega}{\Omega^2 + \phi}, \quad \text{and for dispersion jump of phase velocity:} \]

\[ \Delta = \frac{\Omega}{\Omega^2 + \phi} \]

Note, that in the case \( \frac{\Omega}{\Omega^2 + \phi} = 0 \), the expression (4) is analytically equivalent to dispersion low for Lorentz's model, and for the case \( \frac{\Omega}{\Omega^2 + \phi} = 0 \) is equivalent to the one for the Debye's model.

With use of Efros's theorem about generalized convolution, the following new analytical representation of the Green's function (2) with wave number (4) is obtained:

\[ K’(p) = \frac{p}{c_\infty} \left( 1 - \frac{p}{c_\infty} \right) \left( \frac{\Omega}{\Omega^2 + \phi} \right) \]

The representation (5), (6) is pure real. The elastic (delta-function) precursor and high-frequency Sommerfeld's precursor (for \( A > 0 \)) are extracted in analytical forms in this representation.

The relatively low-frequency component,
corresponding to Brillouin's precursor is expressed in a form of real integral in finite limits. Thus, the obtained expressions allow us to analyze the local pulse propagation in or whole region of a sample. We analyze the dispersion properties of a single resonant relaxation process. The type of pulse propagation is determined in this case by two values: $\Omega \tau$ and $\phi$. In dependence on these values the phase velocity and attenuation coefficient are described by single resonant relaxation process.

The following expressions for the attenuation coefficient and phase velocity can be obtained from (4):

$$\alpha(\omega) = \frac{\Delta \rho}{2\omega_0^2 \tau},$$

$$c(\omega) = c_\infty \left(1 - \frac{\Delta \rho}{2}\right).$$

The features, which the attenuation coefficient and phase velocity can have at various values of $\Omega \tau$ and $\phi$, for stable, dissipative media, are shown on Fig.1 and Fig.2. The attenuation coefficient can monotonically increase with frequency or can have maximum. The phase velocity also can be monotonic, or it can have maximum or minimum or the both together.

On the plane $\Omega \tau - \phi$, the domains, in which the attenuation coefficient and phase velocity have different features, and which correspond to different pulse dynamics, are shown on Fig.3.

A $> 0$ in domain 1, 2, 3 and 4
A $< 0$ in domain 5 and 6
The analysis of pulse velocity and attenuation coefficient and direct numerical calculations of pulse dynamics allow us to image the following picture of different types of pulse dynamics for different domains of values of parameters $\Omega \tau$ and $\phi$.

The pulse dynamics for parameters values on the line $\Omega \tau = 0$ correspond to the pure relaxation Debye's model. In that medium the pulse body of gaussian form with power law of decay (Brillouin's precursor) follows on a monotonically decreasing curve of the phase velocity, where the Brillouin's precursor also has a gaussian form with exponential decay. In domains 1 and 3, where $\Omega \tau$ has a maximum, the Brillouin's precursor begins to oscillate. It is noted that all variation of pulse dynamics occurs smoothly with variation of parameters. In domains 5 and 6, where phase velocity has a minimum, the Brillouin's precursor begins to oscillate. Need to note, that all variation of pulse dynamics occurs smoothly with variation of parameters. In domains 5 and 6, where phase velocity has a maximum, and high-frequency components propagate with phase velocity greater than $c_\infty$, that leads to appearance of Sommerfeld's precursor propagating with pulse front. The oscillation frequency of Sommerfeld's precursor increases with passed distance and its amplitude decreases exponentially. In the limit case, corresponding to the Lorentz's model ($\phi = \pi/2$), the exponent index vanishes and the power gross of the precursor appears. More low frequency part of spectrum is responsible for formation of Brillouin's precursor, which is related to the Lorentz's model. The qualitative behavior of pulse dynamics for parameters belong to domains 5 and 6 is shown in Fig. 5.

Above the features of short pulse dynamics in stable, dissipative media with positive attenuation coefficient in whole frequency domain were considered. However, the obtained solution (5), (6) proves to be correct and for non stable, active media on the beginning stage of pulse propagation, while its amplitude is enough small and it is possible to use the linear approximation. In this case the frequency dependencies of attenuation coefficient and phase velocity are more variety as it is shown on Fig. 6, Fig. 7.
Fig. 6. The attenuation coefficient in dependence on dimensionless frequency for different values of \( \Omega \tau \) and \( \phi \).

Fig. 7. The phase velocity in dependence on dimensionless frequency for different values of \( \Omega \tau \) and \( \phi \).

On the plane \( \Omega \tau - \phi \) the domains, in which the attenuation coefficient and phase velocity have different features corresponding to different pulse dynamics, are shown on Fig. 8, Fig. 9 for arbitrary active or passive media.

Fig. 8. The domains of parameters \( \Omega \tau \) and \( \phi \), corresponding to different behavior of phase velocity (the region numbers correspond to number of curves on Fig. 7).

Fig. 9. The domains of parameters \( \Omega \tau \) and \( \phi \), corresponding to different behavior of attenuation coefficient (the region numbers correspond to number of curves on Fig. 6).
As seems on Fig. 6, Fig. 7, for pure active media 5, 6, for which the attenuation coefficient is negative in whole frequency domain, its frequency dependence, as well as frequency dependence of addition to phase velocity, differ only by sign from pure dissipative media 1, 2. For that media the basic forms of pulse dynamics will be determined by frequencies with maximal amplification coefficient. At this, if the maximum is achieved at high frequencies, then the pulse dynamics will be determined by Sommerfeld's precursor, if the maximum is achieved at finite frequency, then the pulse dynamics will be determined by Brillouin's precursor.

However, the most interesting case, as it seems, is the case of active-dissipative media 3, 4, 7, 8, when on some frequencies medium is amplified, but on another frequencies it is dissipative. There are two basic regimes: amplification at low frequencies and dissipation at high frequencies 7, 8, and versus 3, 4. The pulse dynamics for two last cases is shown on Fig. 10, Fig. 11. As seems on the figures, the pulse behavior is determined by frequencies corresponding to the maximum of amplification coefficient.

References