Abstract. The stabilization of an optical pulsating discharge in a supersonic flow for the first time realized in [Tretyakov P.K. et al. //Physics-Doklady.- 1994, v.39, No.6; and 1996, v.41, No.11] gave new possibilities to simulate and study experimentally gas flows with an external energy supply. In addition to experimental data on the wave structure, the work presents the results of an analytical study of energetic and gasdynamics parameters correlation, allowing one to estimate effects of laser plasma/supersonic flow interaction in a broad range of initial conditions.

Two different quasistationary flow conditions formed by proper laser beam focussing are studied herein. One regime is characterized by the limited rate of energy supply behind a light-supported detonation wave, the other one, by instantaneous energy deposition (point explosion).

The dependence of gasdyanmic parameters and thermal wake boundaries on the free-stream flow Mach number and laser power was determined. For example, the relative values of the total pressure different for both regimes at the same power absorbed decrease suffitiently, more than by an order of magnitude, with Mach number increasing between 1-10. Besides, as integrated effects depend on the body and thermal wake dimention ratio, these effects are strengthened by increasing power up to certain limits.

1. Introduction

Stabilization of a pulsating optical discharge in a supersonic flow first achieved in [1-3] offered wide potentialities for experimental simulations and studies of flows with external energy supply. The problem of achieving nearly steady (quasistationary) energy deposition into the flow was solved (by ITAM in cooperation with Institute of Laser Physics, SD RAS) using a pulsed-periodic radiation emitted by a CO2-laser with a mean output energy of 1 to 2.5kW at a repetition frequency $f=12.5-100$kHz. The power density of each focused pulse is greater than the threshold value for the breakdown (over $10^8$ W/cm$^2$), which provides stabilization of the pulsating plasma in an argon supersonic flow. Experimental data showed that, at a high frequency, the pulsating plasma thermal source generates a quasistationary wave structure consisting of a shock wave in its nose part and of a thermal wake spreading far downstream [2-6]. On the basis of available laser plasma dynamics concepts [7,8] for the region behind the light-supported detonation wave (LDW), flow parameters in the thermal wake were also determined [9] under condition of large length $l$ of the breakdown plasma (with respect to its diameter $d$). In experiments the above indicated condition was ensured by proper focusing of the laser radiation. In the case of short focusing, i. e., at $l\sim d$, the conditions for light-supported detonation wave are lacking and the character of the energy deposition processes suits better the condition for an instantaneous local release of energy. The latter condition is usually used in numerical studies [10,11] of interrelation between space-time energy parameters of the heat source and the flow structure in the vicinity of streamlined bodies. An analytical model of an intense point explosion was also used in [12] for predicting desired configuration of the shock wave that can be initiated by a series of laser pulses. As shown below, the potential of the analytical approach within the limits of this model can be substantially widened. Indeed, apart from the space-time scales of the process, it becomes possible to find correlation between its gasdyanmic and laser radiation energy parameters using generalized results of numerical studies of a point explosion with allowance for counter pressure [13,14].

The purpose of the present work is to perform a comparative analysis of the wave structure and thermal wake parameters in a supersonic flow with a pulsating plasma thermal source for two utmost focusing conditions for the laser beam (either extended at $l>>d$ or local at $l\sim d$). This analysis is based on a developed approximate method for evaluation quasistationary parameters behind the breakdown region in dependence on frequency-energy characteristics of the laser radiation and on the flow Mach number. To reveal specific features of the two flow conditions and differences between them, the comparative analysis is carried out at identical mean power of laser radiation and initial conditions of the experiments performed (at $l>>d$).

2. Extended plasma heat source. Physical models used and assumptions adopted.

Among known regimes of discharge propagation along a laser beam [7] in the case of its sharp focusing, only the light-supported detonation regime can be operative [15]. The light emitted by
laser is absorbed in a thin layer behind the front of the LDW with a certain characteristic absorption length smaller than the beam diameter (the condition for the existence of an LDW). The propagation velocity \( V \) of the LDW front is given by the following equation [7]:

\[
V = \left[ 2(\gamma^2 - 1) \frac{W}{s p} \right]^{1/3}
\]

where \( \gamma \) is the ratio of specific heats, \( W \) is the radiation power (in the pulse), \( s \) is the beam cross-sectional area, and \( p - \) the initial gas density. In the power-density range \( W/\rho = 10^3 - 10^6 \text{W/cm}^2 \) typical of LDW conditions this velocity is hypersonic (several \( \text{km/sec} \)). The flow parameters behind the LDW front (denoted with the subscript \( W \)), namely: the density, pressure, internal energy (or enthalpy), flow velocity and velocity of sound, are given by the following well known relations for detonation waves:

\[
\begin{align*}
\rho_W &= \rho_\infty \left( \frac{W}{\rho W} \right)^{1/(1+\gamma)} \\
P_W &= \rho_W V^2 = 1/(1+\gamma) \\
e_W &= c_W = \gamma V/(\gamma+1) \\
u_W &= c_W = \gamma V/(\gamma+1)
\end{align*}
\]

Here the subscript \( \infty \) refers to the free-stream parameters. In the optical breakdown plasma with a high degree of ionization (~1) and temperature of about 25000K, the apparent adiabatic index is expected to lie in the range 1.15-1.25 with an average value \( \gamma = 1.2 \). Under the experimental conditions of [2,3], at a repetition frequency of laser pulses 100kHz \( V = 5.7 \text{km/sec} \), the pressure behind the LDW front is \( P_W = 34 \text{MPa}, \quad c_W = 40 \text{MJ/kg} \), and the argon temperature is 24500-25000K (and \( \gamma = 1.21 \)). The peak values of the above-indicated parameters are attained in a narrow zone the length of which is no greater than the laser beam diameter, 0.2-0.25 mm.

The pulsed-periodic regime of energy supply is characterized by rapid (the pulse duration is \( \tau << t = 1/f \) formation of an extended plasma, which is removed downstream over the distance \( l = u t \) until the next breakdown begins. The measured lengths of the glow region are about 5-8 mm, in line with the values predicted by the formula \( l = \frac{V}{l} = \frac{V}{t} \) from the measured power dynamics of laser pulses \( W(t) \) (or respective average values of \( W \) and \( t \)). Visualization data showed that, as the repetition frequency of the pulses increases (in excess of \( \sim u/l \)), a continuous and extended thermal wake was formed behind the breakdown region. The nonstationary shock waves generated by each breakdown give rise to a quasistationary shock wave around the nose part of the thermal source as it happens in a flow around solid body. The threshold frequencies given by the relation \( f_1 = 1/t_1 = u/l \) for the experimental conditions with repetition frequencies of laser pulses \( f = 12.5, 25, 45, \) and 100kHz are shown in Fig.1 as normalized values \( f/f_1 \) for the range \( \gamma = 1.15-1.25 \). The main result is the following: the working and threshold frequencies coincide (i.e., \( f/f_1 = 1 \)) in the frequency range 45-65kHz in which the measured decreasing relative values of the aerodynamic drag \( C/D \) of streamlined models attains an almost constant value. In view of the fact that the range of mean radiation power is rather narrow, this result confirms that the condition for the transition to the quasistationary flow regime has the form \( V = u/f_1 \) or \( f_1 = u/l \).

The problem of determining quasistationary flow parameters in the thermal wake can be solved using the well known results of numerical and experimental studies [8,16] of breakdown plasma dynamics. In the coordinate system attached to the LDW, the plasma flow detaches from the front with the local velocity of sound. Therefore, the flow pattern corresponds to an underexpanded sonic nozzle outflow of expanding plasma into a co-current hypersonic flow. The interaction between the two flows results in appearance of a quasistationary shock wave, and the heated gas expands and gets acceleration in the axial direction, forming a high-temperature jet ranging over a distance up to several hundreds of the “nozzle” diameter. In this case, as the numerical study [8] showed, the radius of the shock wave and the pressure distribution in the asymptotic region (over a distance far in excess of the diameter of the energy-releasing region) can be determined from the point explosion model. Simultaneously, the high-temperature gas flow in the flow core (jet) with good accuracy is isentropic owing to weak intensity of oblique shocks in the internal region of the flow. Thus, determination of the pressure distribution behind the optical breakdown region is the main problem. Its solution should permit determination of other flow parameters in the thermal wake also.

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[Image 316x492 to 525x688]

Fig.1. Experimental data on aerodynamic drag reduction \( C/D \) and calculated normalized frequency \( f/f_1 \).
We used the solution of the problem about point explosion with allowance for counter-pressure [13] for a cylindrically symmetrical geometry with ignored edge effects since the diameter of the energy-releasing region is far smaller than the characteristic spatial scale.

According to the adopted model, the radius of the wave \( r(x) \) at a distance \( x \) from the LDW front was determined at the time \( t = x/V \) after explosion of a linear charge with a specific (per unit length) energy \( E_0 = \text{Ws} \). Figure 2 shows the distributions of the relative pressure \( P_0/P_\infty \) in the central part of the flow at different moments. The vertical line 1 corresponds to the beginning of a laser pulse \((P_0 = P_\infty \text{ at } t=0)\), pressure profile 2- to its end \((t=t_{\tau}=1.1\mu sec)\), and profile 3 - to the beginning of the next laser pulse \((t = t_{\tau_2}=10\mu sec \text{ for } f=100kHz)\). The axis \( x \) is in the direction of a gas flow and laser beam, the latter being focused at \( x=6.3\text{mm} \) \((r=0)\). The LDW propagates in the counter direction, and at \( t=\tau \) \( x=0 \). The dashed lines continue the respective solution into the expanded region of \( x \) and \( t \); they show the spatial profiles of pressure for the conditions of stationary radiation at the above indicated moments. In reality, the breakdown region is limited by the range \( x=0-6.3\text{mm} \) \((=V\tau)\); it is why the actual pressure profiles differ from the predicted ones. Nevertheless, these results give all necessary data on dynamics and characteristic scales of the process in the breakdown region. At a small (several diameters of the laser beam, or \( \sim 1\text{mm} \)) distance behind the LDW front there occurs an abrupt pressure drop from several tens of MPa down to a level comparable with a pressure in the flow \((56kPa)\). At every position of the LDW, the pressure profiles differ more weakly from one another as we moves farther from the breakdown region; these profiles come close together at a distance no greater than the length of the breakdown zone \( l \), where the flow becomes isobaric. Thus, a certain quasistationary pressure distribution weakly nonuniform along the downstream direction (and, hence, similar distributions of other parameters) is attained behind the nonstationary region of the optical breakdown. The repeated breakdown in a cold gas with “entrained” plasma maintains the steady flow regime. The glow behavior is indicative of rapid transition from nonstationary flow conditions to stationary ones (see photo in Fig.2); here, the bright region (breakdown plasma) is followed by a region with weakly varying glow intensity in the thermal wake.

The question about the magnitude of the quasistationary pressure can be solved by comparing visualization data and the natural glow of the wake, on the one hand, and the predicted radius \( r(x) \) at various times from the beginning of laser pulse, on the other. In figure 3, these solutions are shown (by curves 1–3) for three (analogous to Fig.2) moments: \( t=0 \), \( \tau \), and \( 1/f \) after the breakdown. The experimental data are close to curve 2, i.e., for \( t=\tau \), when the maximum amount of gas is "entrained" into the motion. It is why the pressure profile at the end of laser pulse was used to calculate other wake flow parameters. Quantifying the reduction of pressure with the parameter \( p = P_0/P_\infty \) and using formulas for an isoentropic flow, we immediately obtain the following relations for the relative enthalpy and density in the central part of the wake flow:

\[
h_0/h_\infty = p^n, \quad \rho_0/\rho_\infty = p^{\gamma/(\gamma-1)n} (1+\gamma)/\gamma,
\]

where \( n = (\gamma-1)/\gamma \).

The flow velocity and the Mach number are obtained by similar manner:

\[
u_0/V = (1+2(1-p^\gamma)/(\gamma-1))^{1/2} \gamma(1+\gamma) + u_\infty/V,
M = (\nu_0/V - 1)(1+\gamma)\gamma p^{n/2},
\]

where \( u_\infty \) is the main flow velocity.
Figure 4 shows the distributions of the relative enthalpy (a) and total pressure (b), and also the distribution of the thermal wake Mach number $M_0$ for the conditions of a main argon $M_\infty=2$ flow. In the region occupied by the quasistationary flow, the enthalpy amounts to 0.4 of its extreme (behind the LDW front) value, which corresponds to a temperature drop from 24500 K to 12500 K. It is essential that the gas density turned out to be diminished by two orders of magnitude. Owing to high local velocity of sound, the Mach number is relatively low, $M_0=1.4-1.6$ at high flow velocity. The interrelation between these parameters results in a decrease in the total (stagnation) pressure $(P_0)^*$ with respect to its value $(P_\infty)^*$ in the main flow.

![Graph](image)

Fig. 4. Flow parameters axial distribution in a quasistationary thermal wake.

In the frame of this model, it is possible to evaluate the effects due to flow non-one-dimensionality caused by the radial divergence of the flow and plasma radiation losses. The first effect can be estimated like in [7] by introducing an additional parameter $\delta$ and using in the calculation, instead of $W/s$, the effective value $\delta(W/s)$. In the experiments, the power density was little more than its threshold value; therefore, the value $\delta=0.5$ was used to obtain a rough estimate. The results are shown in Figs.2-4, 7 with respective curves 2*. The change in the flow parameters is relatively small owing to the fact that the dependence $V \sim W^{1/3}$ is weak and the decrease in $P_\eta$ is small.

The radiation losses can be estimated from the ultimate value of the total radiant flux $S_\eta(x)=4\pi \varepsilon_\eta(x,n_e,n_I)dx$ of the breakdown plasma extending over the distance $x$ from the LDW front. An approximate expression for the integral (over spectrum) emission capacity $\varepsilon_\eta$ of dense argon plasma was obtained in [16] when studying a continuous optical discharge. Also used was the predicted distribution of parameters behind the LDW front. Comparison between the radiant flux with the mass energy flux $S_\eta=\rho_0Vh_W$ permits gaining a better insight into the contribution of the radiation energy losses to the total energy balance. Under the experimental conditions, $S_\eta=6.3 \cdot 10^7$ W/cm², which well correlates with the laser radiation flux $W/s=(0.5-1) \cdot 10^8$ W/cm² that provides for the heating and the gas motion in the breakdown region. The predicted distribution of the integral emission capacity of argon plasma behind the LDW front displays an abrupt decay (by three orders of magnitude) over a distance comparable with the beam diameter. It is this region that contributes predominantly $(2.2 \cdot 10^6$ W/cm²) to the total radiant flux $S_\eta=2.24 \cdot 10^8$ W/cm² throughout the entire length (6.3 mm) of the breakdown region. Hence, $S_\eta/S_\eta=0.036<<1$ and, consequently, the effect due to radiation losses is negligible.

![Graph](image)

Fig. 5. Gasdynamic parameters behaviour in central part of point explosion.

Thus, the obtained solutions show that the quasistationary thermal wake is a high-temperature rarefied supersonic flow. An indirect indication for this is provided by the photographs (in Fig.4) of the flow around pressure gauge, which indicate the presence of a compression shock in front of them. The large extension of the wake (no less than 150d) observed in experiments gives grounds for ignoring
3. Point source of heat in a supersonic flow.

In contrast to an extended source, here we assume that the energy is absorbed instantaneously at the point where the radiation is focused, i.e., the main assumptions of the point explosion model are assumed fulfilled. Assuming also that the repetition frequency of laser pulses is sufficient for initiating a quasistationary flow (this condition is given below), we determine the initial energy parameter (energy per unit length) \( E^0 = N/u_\infty \). Here \( u_\infty \) is the main flow velocity and \( N \) is the measured mean power of the absorbed laser radiation \( N = W t_\tau \), which gives \( N = 1.6 \text{ kW} \) for \( f = 100 \text{ kHz} \). The value \( E^0 = 4 \text{ J/m} \) obtained for these experimental conditions characterizes the space-time scales of the point explosion of cylindrical symmetry [13]: 

\[
p^r = (E^0 / P_\infty)^{1/2} \quad \text{and} \quad \rho^r = (P_\infty / P_\infty)^{1/2} \quad (8.4 \text{ mm and } 54\mu\text{sec}).
\]

In the analytical consideration of the radial distribution of parameters at different moments after the explosion, tabulated data on gasdynamic parameters of [14] were used. These data are represented as functions of dimensionless variables \( \xi \) and \( q \), where \( \xi = (r_t / r_0)^2 \), \( r_t \) and \( r_0 \) are the radii of the points of interest and shock wave; \( q = (a_0 / c)^2 \), \( a_\infty \) is the velocity of sound in the free stream, and \( c \) is the velocity of the shock wave. For a fixed distance \( x \) from the point where the explosion happens \( t = x / u_\infty \), and the dimensionless parameter \( t / \theta \) determines the value of \( q \).

Figure 5 shows the variation of the relative pressure \( P_\infty / P_\infty \), velocity of sound \( a_0 / a_\infty \) and radial (expansion) velocity of medium \( \dot{v}_t / \dot{v}_\infty \) (1 and 2 for \( \gamma = 1.3 \) and 1.67) in the central part \( (x = 0) \) of the flow with increasing \( q \) (time after explosion). The same figure shows the ranges of \( q \) for the flow Mach numbers \( \text{M}_\infty = 2 \) and 10 in the interval \( x = 5-30 \text{ mm} \). The common feature of these solutions is that \( q > 0.35 \), of almost constant (within several per cent) values of \( P_\infty / P_\infty \), \( 1 \), \( a_0 / a_\infty \), \( 5 \) and \( v_t / v_\infty \), (and the change of the sign of the velocity at \( q = 0.6-0.8 \)). The isobaric quasistationary flow is determined by the coordinate \( x = u_\infty (t / \theta)^{1/2} \) (where \( t / \theta \approx 0.11-0.13 \), depending on \( \gamma \) for \( q > 0.35 \)).

The radial profile of the relative density \( \rho / \rho_\infty \) in the argon flow with \( \text{M}_\infty = 2 \) (it is shown for \( \gamma = 1.3 \), the behavior for other \( \gamma \) is similar) for two fixed values of \( q, q = 0.6 \) and 0.8 \((x = 7.6 \text{ mm} \) and \( 24.4 \text{ mm}, \) respectively), are shown in Fig.6. The behavior of this parameter allows one to obtain the following estimate of its mean value in the central region: \( \rho_0 \approx 0.03-0.04 \rho_\infty \) (the estimated velocity of sound is \( a_0 \approx 5 a_\infty \)) at the point \( r_0 \), the nearest one to the axis, thus eliminating the singularity of the point explosion theory solution at \( r = 0 \). This choice is also justified by the fact that the energy release takes

dissipative processes caused by, say, vortex formation processes in mixing flows, at least, on the initial stage of development. The validity range of this condition can be clarified by measuring the radial distribution of the total (also static) pressure in the flow behind the plasma thermal source. By now, no systematic studies of this matter have been carried out. Only limited data are available obtained by measuring the total pressure with a Pitot tube 1.8 mm in diameter installed behind the breakdown region (18 and 28 mm). These data for close to quasistationary conditions \((f = 35 \text{ kHz})\) are shown in Figs.6 and 7; they agree well with the predicted values.

![Fig.6](image6.png)

**Fig.6.** Wave structure and radial density distribution in supersonic \( \text{M}_\infty = 2 \) flow caused by point \((x = 0)\) energy deposition.

![Fig.7](image7.png)

**Fig.7.** Results of the comparative analysis: relative total pressure and Mach number \( \text{M}_\infty \) in thermal wake in dependence on main flow Mach number \( \text{M}_\infty \):

I - extended plasma thermal source \((l > > d)\),

II - point explosion model \((l \sim d)\)
place at the spatial scale $d < r_0$. The inner and outer boundaries of the characteristic low density (and high temperature) region are shown below the $x$-axis by lines $r_0$ and $r_1$. With increasing $x$, the first boundary remains nearly unchanged ($r_0$ is smaller than $0.9–1.2$ mm), whereas the radius of the outer boundary appreciably (by several times) increases, clearly showing the expansion of the region with a nearly constant density gradient. The latter is a feature that distinguishes the process under study from the energy release behind the LDW front, where the thermal wake boundary (curve 3) is a weak shock that separates the high velocity plasma from the main stream. Also shown in this figure are the predicted ($r_n = 1.2$ for both values of $\gamma$ 1.3 and 1.67) and measured (points) configurations of the bow shock wave. This shock wave results from superposition of unsteady waves generated by different breakdowns; it is nearly independent of focusing conditions for $r_n > d$.

The dependence of flow parameters on the power $N$ manifested itself in the interrelation between $E^0, \rho^0$, and $f$, which shows that the latter quantities increase proportionally to $N^{1/2}$. Simultaneously, the values of $\nu_0/\rho^0$ and $q$ decrease, whereas the radius of the shock wave and the gasdynamic characteristics grow in value. Also shown in Fig. 6 (with arrows $10 \gamma f$) is evolution of $r_0, r_1$ and $r_n$ with a tenfold increase in the power. Most pronounced is the increase in $r_0$ (by a factor of 2.5-3) and less pronounced is that in the shock wave radius. Thus, the effect due to increasing power is manifested in the enlargement of the low density region with earlier attainment of the quasistationary values of flow parameters ($q > 0.35$).

The condition for the formation of a quasistationary flow acquires the form $x = r_0(x)$ (or $r_1(x)$). Graphic solution is a cross point of corresponding lines in Fig. 6, then $f = u_0/r_0$. As compared to the case of an extended source, for which $u_0/f - 1$ at a short beam focusing a higher (by factor $l/r_0$) frequency is required. For the less stringent requirement $u_0/f - r_n$ (here, spherically symmetrical solutions should be used) the value of $f$ turns out to be several times ($r_0/r_0$) decreased; however, here again the spatial non-uniformity of flow parameters becomes more pronounced. In both regimes of energy supply, the required frequency increases with increasing flow velocity (Mach number).

4. Results of the comparative analysis.

To determine the mean values of velocity characteristics of the quasistationary flow (such as Mach number or total pressure), we used the following general property of the solution for the expansion velocity of medium in the central part of the explosion: $v_f/a_u = 0$. This means that in the region occupied by the isobaric flow the mean axial velocity of the medium $u_0$ at the center of the flow (i.e., in the region of radius $r_0$) is close to the main flow velocity, i.e., $u_0 = u_u = M_u a_u$. The velocity of sound under these conditions is nearly constant: $a_0 = K a_u$ with the coefficient $K = 5$ (within 10% for different $\gamma$). As a result, we determine the Mach number in the central part of the flow: $M_v = M_u/5$. This result is compared in Fig. 7 (curve II) with the data (curve I) obtained for an extended energy source (with identical initial conditions and the mean radiation power), also in the range where the isobaric condition holds. These data show that in the range $M_u = 1\cdot 10$ the Mach number of the flow in thermal wake $M_v$ is always lower than that in the main flow; it is the lowest one in the regime with explosion for which the flow for $M_u < 5$ is subsonic.

From the known correlation between the gasdynamic parameters in the flow, we have also determined the relative profile of the total pressure $(P_u/P_u)^*$ for the two quasi-stationary energy supply conditions. The results are shown in Fig. 7. Here, the frequencies may differ appreciably from each other. For the first time it is shown that, for identical mean values of power, the stagnation parameters differ considerably (they are three times smaller at short focusing and at low $M_u$). Simultaneously, their values decrease markedly throughout the whole range explored (by more than ten times) with increasing main flow Mach number. The established, for the case under study, steeper variation of the ratio $(P_u/P_u)^*$ at low $M_u$ implies that the effects due to energy supply (for instance, in the flows around bodies), with all other conditions kept unchanged, should change only slightly in the velocity range from $M_u = 5$ on. Besides, since the integral effects due to the energy supply are also determined by the relationship between the crossflow dimensions of streamlined bodies and those of the region in which the flow parameters change appreciably (of a radius smaller than $r_1$), these effects become more distinctly pronounced with increasing power only up to a certain limit, which depends on the ratio between the above dimensions.

5. Conclusion.

In contrast to a point (or by volume) explosion the plasma behind a light-supported detonation wave front gains high velocity in the direction of a main flow. For all that, there is the same wave structure by formal resemblance, including a bow shock wave and a thermal wake flowdown in a supersonic flow with energy deposition. The bow shock wave configuration is
identical for both regimes because it is shaped by superpositions of nonstationary shock waves caused by each optical breakdown; but the thermal wake's boundary dynamics have the difference. To a greater degree both regimes differ in velocity parameters (Mach number, total pressure).

The approach used herein may be developed and adopted for analysing the effects of a local energy deposition in a supersonic flow (for determination of a wave structure and estimation of flow parameters) with application of other kinds (and methods of supply) of radiation energy. It takes to know general mechanisms (models) only, which determine the plasma expansion in a region of energy absorption.

References