Introduction

A development of computing models of self-consistent electro-dynamic and thermo-gasdynamic processes is a significant component of projects on creation of prospective systems of flow control in applied aerodynamics based on weekly-ionized gas technologies. Numerical and experimental researches executed recently indicate to their high efficiency [1-7]. These models can successfully be used not only for study of separate physical processes, but allow also to predict parameters of real aerodynamic objects. The good conformity of the calculated and experimental data allows hope that at the further development of these computing models they can partly replace expensive experiments (at least, at the stage of preliminary designing).

The paper presents preliminary results of study of two-dimensional computing model of self-consistent thermo-gasdynamic and electro-dynamic processes in gas discharge channels with subsonic flows. The \((k-\varepsilon)\) turbulence model and external magnetic field are included into the numerical simulation model. It allows investigating influence of the initial turbulence and external magnetic field on gasdynamic and electro-dynamic structure of gas discharge channels. Schematic of the problem under consideration is shown in Fig.1.

Governing equations

Gas dynamic and gas discharge processes in a gas discharge channel are described by the following system of equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 , \tag{1}
\]

\[
\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \mathbf{V}) = -\frac{\partial p}{\partial x} - \frac{2}{3} \frac{\partial}{\partial x} \left( \mu \nabla \cdot \mathbf{V} \right) + \nabla \cdot \left( \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) + 2 \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) , \tag{2}
\]

\[
\frac{\partial \rho v}{\partial t} + \nabla \cdot (\rho v \mathbf{V}) = -\frac{\partial p}{\partial y} - \frac{2}{3} \frac{\partial}{\partial y} \left( \mu \nabla \cdot \mathbf{V} \right) + \nabla \cdot \left( \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) + 2 \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) , \tag{3}
\]

\[
\frac{\rho c_p}{\partial t} \frac{\partial T}{\partial x} + \frac{\rho c_p V \nabla T}{\partial y} = \nabla \cdot (\lambda \nabla T) + q , \tag{4}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) - D_{\alpha} \nabla n = n_i \frac{E}{N} - \beta n^2 , \tag{5}
\]

\[
\nabla \mathbf{j} = 0 , \quad \mathbf{j} = n_i \mathbf{V} \nabla \varphi - D_{\alpha} \nabla n , \tag{6}
\]

\[
\mathbf{E} = -\nabla \varphi , \tag{7}
\]

where: \(\rho, p\) are the density and pressure; \(u,v\) are the components of the velocity \(\mathbf{V}\) along axes \(x\) and \(y\); \(\mu\) is the effective viscosity; \(T\) is the temperature; \(c_p\) is the heat capacity at constant pressure; \(\lambda\) is the effective heat conductivity coefficient; \(q\) is the heat release power (in the given case – due to the Joule heating); \(n\) is the concentration of charged particles (ions and electrons, \(n_i = n_e = n\)); \(\nu\) is the ionization coefficient; \(\beta\) is the recombination coefficient; \(D_{\alpha}\) is the ambipolar diffusion coefficient; \(N\) is the volume concentration of the neutral particles; \(\varphi\) is the potential of the electric field; \(\mathbf{E}\) is the intensity of the electric field; \(E = ||\mathbf{E}||\); \(\mu_e\) is the mobility of electrons.
For calculation of the effective viscosity and thermal conductivity the \((k-c)\) turbulence models is used:

\[
\mu = \mu_m + \mu_t, \quad (8)
\]

\[
\mu_t = C_\mu \rho k^2 / \varepsilon, \quad \lambda_t = \mu_t c_p / Pr_t, \quad (9)
\]

where: \(\mu_m, \lambda_m\) are the molecular viscosity and thermal conductivity; \(Pr_t=0.7\).

The following system of equations is used for calculation of \(k\) and \(c\) functions:

\[
\frac{\partial k}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \mu}{\partial \rho} \right) = P - \rho e, \quad (10)
\]

\[
\frac{\partial e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \mu}{\partial \rho} \right) = (C_1 P - C_2 \rho \varepsilon) \frac{\varepsilon}{k}, \quad (11)
\]

\[
P = \mu_t \left[ \frac{2}{3} \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \frac{2}{3} \left( \frac{\partial w}{\partial r} \right)^2 - \frac{\lambda_t}{\rho t} \left( \frac{\partial \rho}{\partial r} \frac{\partial \rho}{\partial r} + \frac{\partial \rho}{\partial \theta} \frac{\partial \rho}{\partial \theta} \right), \quad (12)
\]

\[
C_n=0.09; \; C_1=1.44; \; C_2=1.92; \; \sigma_t=1.0; \; \sigma_c=1.3.
\]

\[
C_n = C_{n,0} f_n(Re), \quad C_{n,0}=0.08, \quad f_n(Re)=\exp \left( -\frac{2.5}{1+0.02 Re_f} \right), \quad (13)
\]

\[
Re = \frac{\rho k^2}{\mu e}.
\]

The constitutive relationships for gasdynamic equations \((1)-(4)\) are formulated for molecular nitrogen \((N_2)\):

\[
\mu_m = 2.67 \cdot 10^{-5} \sqrt{M_A T} \frac{1}{\sigma^2 \Omega^{(2,2)}}, \quad \lambda_m = 3.06 \Omega^{(2,2)} T^{0.1472},
\]

\[
T^* = \frac{T}{e/k}, \quad e/k=71.4;
\]

\[
\lambda = (0.115 + 0.354 \frac{c_p}{R_0}) \cdot 8.333 \times 10^{-4} \sqrt{\frac{T}{M_A}} \frac{1}{\sigma^2 \Omega^{(2,2)}},
\]

is the molecular part of the effective heat conductivity coefficient

\[
\lambda_m = \lambda_m + \lambda_c, \quad c_p = 8.317 \frac{7}{2 M_A}, \quad \rho = \frac{p M_A}{T R_0}. \quad (10)
\]

The constitutive relationships for electrodynamic equations are:

\[
u_r = \left( \frac{\alpha}{p} \right) p E \mu_e(p^*), \quad (14)
\]

\[
\mu_e(p^*) = 1450 \frac{1}{p^*}, \quad \beta=2 \times 10^{-7} \text{cm}^3/\text{s}, \quad T_e=11610 \text{K},
\]

\[
N=9.54 \times 10^{19} \frac{T}{p} \quad \text{is the concentration of the neutral particles; } \mu_i \text{ is the mobility of ions.}
\]

The constitutive relationships must also provide calculation of a heat release in a gas due to the Joule heating

\[
q = \eta (j \cdot E) = 1.6 \times 10^{19} \eta [n E^2 (\mu_e + \mu_i) + (D_e - D_0) \text{Grad} n], \quad (15)
\]

and the effect of a magnetic field on gasdynamic structure

\[
F_b = \chi |j| B, \quad \text{where } \eta \text{ is the efficiency of heating } (\eta=0.1 \pm 0.3), \chi \text{ is the efficiency of momentum transfer } (\chi=0.01 \pm 0.5).
\]

**Boundary and initial conditions**

The boundary conditions for velocity and temperature of the gas discharge plasma are formulated as follows:

- on the walls of the channel \((y=0, y=H)\):

\[
T=T_w=300 \text{K}; \quad u=v=0; \quad k=e=0;
\]

- in the entrance section \((x=0)\):
\( T = T_w; \ u = u_0; \ v = 1; \ e = e_0; \)

in the output section \((x = L)\) derivatives of all functions on longitudinal coordinate relied equal to zero.

The initial level of turbulent pulsations was varied according to available experimental data. The assumption of equality of dissipation and generation speeds of the pulsations on insignificant distance from entrance aerodynamic lattice was used.

The boundary conditions for charged particles and electrical potential are formulated as follows:

- on the cathode sections \((y = 0)\): \( \frac{\partial n}{\partial y} = 0 \); \( \varphi = \frac{V_k}{E} \);
- on the dielectric surfaces \((y = 0)\): \( n = 10^{-5} n_0 \); \( \frac{\partial p}{\partial y} = 0 \);
- on the anode \((y = H)\): \( \frac{\partial n}{\partial y} = 0 \); \( \varphi = 1 \);
- in the entrance section \((x = 0)\): \( n = 0 \); \( \frac{\partial p}{\partial x} = 0 \);
- in the output section \((x = L)\): \( \frac{\partial n}{\partial x} = 0 \), \( \frac{\partial \varphi}{\partial x} = 0 \),

where \( n_0 = 10^9 \text{cm}^{-1} \text{cm}^{-1} \).

Some density of plasma near to cathode sections and voltage on them are set as the entry conditions. The gas flow is considered as homogeneous in the channel at the beginning of the calculations.

The ballast resistances \( R_k \) (they are identical for each cathode section) and the currents \( I_k \) are set for 1 cm of length along coordinate \( z \), on which two-dimensional process does not depend.

The time-relaxation splitting method is used for solution of the formulated problem.

**Numerical procedures**

An implicit finite volume method of the second order on special variables and the first order on time was used. All equations were represented in elliptical 5-point finite-difference form and were integrated by SOR on line (with Thomas algorithm) method.

United nonhomogeneous calculation grids

\[ \{ q = x_j, y_j; j = 1, 2, \ldots, NJ; p = x_i, y_i; i = 1, 2, \ldots, NI \} \]

were used both for gasdynamic and for electrodynamic equations.

**Numerical simulation results**

All calculations were performed on calculation grid \( NJ = 200, NI = 100 \) for gas pressure \( p = 50 \text{torr} \) and resistances of external circuit \( R_0 = 12 \text{k} \Omega \). Figures 2 and 3 show distributions of thermo-gasdynamic and electrodynamic parameters of the gas discharge channel with turbulent subsonic gas flow. Figure 2 corresponds to calculation case without external magnetic field, and Fig.3 – with magnetic field \( B = 0.01 \text{Tl} \). The rest initial conditions were as following: \( u_0 = 200 \text{m/s}, \ E = 5 \text{kV}, \ k_0 = 0.1 \).

Submitted calculated data allow make a conclusion on appreciable influence of a magnetic field on gas flow structure. In the case under consideration, distributions of the charged particles and current density change slightly at taking into account magnetic field of \( B = 0.01 \text{Tl} \). The maximal temperature in the channel is reduced (Fig.2, c and 3, c). The most important result is the appreciable change in structure of a gas flow near to the cathode surface, namely, a vortical structure of gas flow is observed nearby to the cathode sections. It is established that with increase of the magnetic field induction, the local perturbances of subsonic gas flow grow. We shall notice that calculation results submitted in Fig.2, 3 were received for the factor of an impulse transfer efficiency of \( \chi = 0.1 \), and the factor of energy transfer efficiency of \( \eta = 0.2 \). The further perfection of the calculation model means specification of these factors.

**Conclusion**

A self-consistent 2D numerical simulation model of gasdynamic and electrodynamic processes over a transverse magnetic field in gas discharge channels has been developed.

This model is based on the Reynolds averaged Navier-Stokes equations with additional \((k-\varepsilon)\) model of turbulent mixing in gas flow, and electro-dynamic equations, describing behavior of charged particles in electric and magnetic field.

Preliminary numerical simulation results on electro-dynamic and gasdynamic structure in gas-discharge channels with sectionalized cathodes have been received.

It is shown that magnetic field of \( B = 0.01 \text{Tl} \) has significant impact on electrodynamic and gasdynamic structure of gas discharge channels. The study allows make a supposition that a magnetic field can be used as control factor for partially ionizing gas flows.
**Fig. 2.a.** Concentration of charged particles in gas-discharge channel with sectionalized cathodes

**Fig. 2.b.** Current density in gas-discharge channel with sectionalized cathodes

**Fig. 2.c.** Temperature distribution in gas-discharge channel with sectionalized cathodes
Fig. 2.d. Isolines of the x-axial component of velocity in gas-discharge channel with sectionalized cathodes.

Fig. 3.a. Concentration of charged particles in gas-discharge channel with sectionalized cathodes and with external magnetic field.

Fig. 3.b. Current density in gas-discharge channel with sectionalized cathodes and with external magnetic field.
Fig. 3.e. Temperature distribution in gas-discharge channel with sectionalized cathodes and with external magnetic field

Fig. 3.d. Isolines of the x-axial component of velocity in gas-discharge channel with sectionalized cathodes and with external magnetic field

Fig. 3.e. Velocity field in gas-discharge channel with sectionalized cathodes and with external magnetic field
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References