Abstract. Quasi-one dimensional numerical simulation throughout an engine was carried out for detailed performance analysis. Simple scramjet engine model including inlet, MHD generator, combustor, MHD accelerator, and expansion nozzle was introduced. MacCormack method was employed for unsteady numerical simulations. Maximum combustor exit stagnation temperature was set for the operating condition. Engine performance was evaluated by inlet-to-exit thrust normalized by mass flow rate. Dependency of engine performance on flight Mach number was investigated. MHD generator had successfully reduced the stagnation temperature prior to the combustor which enabled greater energy injection at the combustor. Therefore, by introducing the MHD energy bypass system, scramjet engine was shown to improve its thrust performance at high Mach number conditions. Qualitative agreement with analytical calculation was obtained, although the phenomenon of thermal choking in the MHD generator and the combustor was found as operating limitations. Strong dependence of thrust on channel configurations were also found.

Keywords: Thrust, Mach number, choking, optimum expansion

1. Introduction

One of the most essential components of supersonic and hypersonic flying vehicle is the scramjet engine. In contrast to turbo jet system, simple structure of scramjet system has higher potential in operating conditions, especially at high Mach number conditions. Engine materials could bear higher temperature when it is free from mechanical stress. But scramjet engine also has temperature limitation at hypersonic regime where flow stagnation temperature become excessive. Magnetohydrodynamic (MHD) energy bypass scramjet engine system, known as an AJAX concept, is proposed for fundamental solution to conventional scramjet engine. The energy bypass system could extract portion of energy prior to the combustor by MHD generator and reduce stagnation temperature. Recent studies suggests the feasibility of MHD energy bypass scramjet engine system, although many studies are based on analytical calculations. Analytical calculations are very useful for performance estimations and feasibility considerations. On the other hand, numerical simulations are essential for understanding fluid dynamic characteristics and detailed performance analysis. In this work, quasi-one dimensional numerical simulation through out the engine was carried out to consider the engine characteristics.

2. Numerical Model

2.1 Interaction parameter, Energy addition ratio

The essential feature of MHD flow is the existence of Lorentz force and the Joule heating. Therefore, these effects are needed to be included in the basic equations. To represent Lorentz force, a dimensionless number interaction parameter $s$, defined as follows, will be used.

$$s = \frac{\text{Lorentz force}}{\text{Flow momentum}} \cdot L$$

Interaction parameter is the ratio of Lorentz force to flow momentum. Energy input at the combustor is represented as energy addition ratio $EA$, which is the ratio of inflow energy and added energy.

$$EA = \frac{\text{Input energy}}{\text{Flow energy}} \cdot L$$

Energy addition ratio is also a dimensionless number. $L$ is representative length in both definitions.

2.2 Basic equations

Quasi-one dimensional basic equations used in this work are as follows.

Continuity equation
\[
\frac{\partial}{\partial t} (\rho u A) + \frac{\partial}{\partial x} (\rho u^2 A) = 0
\]  
(3)

Momentum equation

\[
\frac{\partial}{\partial t} (\rho u A) + \frac{\partial}{\partial x} [(u^2 + p) A] = p \frac{\partial A}{\partial x} + F
\]  
(4)

Energy equation

\[
\frac{\partial}{\partial t} (e A) + \frac{\partial}{\partial x} [u (e + p) A] = Q
\]  
(5)

Here, e is the total energy

\[
e = c_s T + \frac{u^2}{2}
\]  
(6)

F and Q are the terms of Lorentz force and Joule heating / energy addition respectively. F and Q varies at each engine components. In the MHD generator,

\[
F = \frac{\rho u^2}{L} s_{\text{MHD}}
\]  
(7)

\[
Q = \eta_{\text{MHD}} \frac{\rho u^2}{L} s_{\text{MHD}}
\]  
(8)

In the combustor,

\[
F = 0
\]  
(9)

\[
Q = \rho u \left( c_s T + \frac{u^2}{2} \right) \frac{1}{L} E A_{\text{comb}}
\]  
(10)

And in the MHD accelerator,

\[
F = \frac{\rho u^2}{L} s_{\text{acc}}
\]  
(11)

\[
Q = \frac{1}{\eta_{\text{ass}}} \frac{\rho u^3}{L} s_{\text{acc}}
\]  
(12)

F and Q are both 0 in other engine components. Here, \(\eta_{\text{MHD}}\) and \(\eta_{\text{acc}}\) are electrical efficiency and acceleration efficiency.

MacCormack method was carried out for unsteady quasi-one dimensional simulation.

Thrust could be described as a sum of increased momentum and pressure difference at the exit.

\[
F = \dot{m}_a (u_{\text{exit}} - u_{\text{inlet}}) + (p_{\text{exit}} - p_{\text{env}}) A_{\text{exit}}
\]  
(13)

Since the thrust strongly depends on mass flow rate, normalized thrust will be used to discuss thrust performance in this work.

\[
\frac{F}{m_a} = \dot{m}_a (u_{\text{exit}} - u_{\text{inlet}}) + (p_{\text{exit}} - p_{\text{env}}) A_{\text{exit}}
\]  
(14)

2.3 Model

Schematic of MHD energy bypass scramjet engine is shown in Figure 1. Scramjet engine channel includes inlet, MHD generator, combustor, MHD accelerator, and expansion nozzle. Portion of energy extracted at the MHD generator is used to run on-board systems or ionizers and the rest is bypassed down stream and used at the MHD accelerator.

![Schematic of MHD energy bypass scramjet engine system](image)

2.4 Calculating conditions

The following conditions were applied to the calculation.

<table>
<thead>
<tr>
<th>Table 1. General calculating conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working gas</td>
</tr>
<tr>
<td>Inlet static pressure</td>
</tr>
<tr>
<td>Inlet static temperature</td>
</tr>
<tr>
<td>Inlet Mach number</td>
</tr>
<tr>
<td>Interaction parameter at MHD generator</td>
</tr>
<tr>
<td>Energy addition ratio at combustor</td>
</tr>
<tr>
<td>Energy bypass ratio</td>
</tr>
</tbody>
</table>

The inflow pressure and temperature correspond to that of altitude 30,000m. Actual scramjet engine utilizes oblique shock waves in front of the engine for flow compression. Therefore not only the velocity but pressure, density, and other parameters vary by flight mach numbers, although it is assumed constant in this work.
The bypassed energy ratio is the ratio of energy extracted at the MHD generator to injected energy at the MHD accelerator. Interaction parameter at the MHD accelerator is adjusted to match the energy bypass ratio.

3. Engine Performance Characteristics

In this section, engine characteristics were determined on fixed channel dimension. Conditions at MHD generator, combustor, MHD accelerator were varied. First, inlet Mach number was fixed to 5 and performance dependency on interaction parameter and energy addition ratio was determined. Then, characteristic on flight Mach number will be discussed.

3.1 Channel dimensions

Schematic of the engine channel is shown in Figure 2. Channel cross sectional area of inlet and MHD generator is constant. Diverging channel was designed to have continuous second derivatives at the area of combustor, MHD accelerator, and expansion nozzle. Channel width is set to 0.25m through out the channel. The results obtained on this channel dimension came up to perform overexpansion, and cause low pressure at the exit.

As it could be known from Eqn.(13), low exit pressure leads to low and / or negative thrust. It is not realistic to have negative thrust during supersonic flight. But in this section, only the qualitative characteristics will be considered. As it is discussed in Section 4, this overexpansion could be improved by optimizing the channel profile.

3.2 Effects of interaction parameter

To consider the effects of the MHD energy bypass system, interaction parameter at the MHD generator and accelerator was varied with constant energy addition ratio in the combustor. Dependence of normalized thrust to the interaction parameter is shown in Figure 3.

In the MHD generator, Lorentz force works against the flow and therefore the interaction parameter at the MHD generator is negative. Hence, left had side of Figure3 is the condition which more energy is extracted at the MHD generator and re-introduced at the MHD accelerator. Each line represents the constant-energy addition ratio conditions.

It could be known that as the interaction parameter decrease and the bypassed energy through MHD system increase, thrust performance decrease. Flow velocity at two conditions of $EA=0.2$, $s_{MHD}=0.0$ and $EA=0.2$, $s_{MHD}=-0.3$ are shown in Figure 4.

Major difference in flow velocity from the MHD generator to the exit could be observed. Even though same amount of energy as the extracted energy is injected at the MHD accelerator, obtained flow velocity at the exit was relatively low. Interaction in the MHD device is not only the Lorentz force, but Joule heating also occurs at the same time since the efficiency of the generator and the accelerator is not ideal. The occurrence of Joule
heating means that the kinetic energy is converted into thermal energy in consequent exhaust momentum and thrust loss.

### Table 2. Energy components at the exit [kJ/m³]

<table>
<thead>
<tr>
<th>Interaction parameter</th>
<th>0.0</th>
<th>-0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic energy $\frac{1}{2}m\dot{u}^2$</td>
<td>3.76</td>
<td>3.56</td>
</tr>
<tr>
<td>Internal energy $\rho c_p T$</td>
<td>0.873</td>
<td>1.07</td>
</tr>
<tr>
<td>Total</td>
<td>4.64</td>
<td>4.64</td>
</tr>
</tbody>
</table>

Table 2 shows kinetic energy and thermal energy at the engine exit. When MHD energy bypass system is applied, total energy has not changed but kinetic energy has decreased. This implies that MHD energy bypass system itself does not increase the thrust. The major advantage of this system is the potential for stagnation temperature reduction. To confirm this potential, stagnation temperature distribution to each condition is shown in Figure 5.

![Fig.5. Stagnation temperature distribution at different interaction parameter](image)

Lower stagnation temperature was obtained by introducing the MHD system. Especially, significant stagnation temperature decreased was realized at the combustor while lower stagnation temperature was obtained at MHD generator, accelerator, and expansion nozzle also. This result suggests the possibility of stagnation temperature reduction in the combustor by using the MHD system. In other words, more energy could be added in the combustor with the equivalent combustor exit stagnation temperature. The advantage of larger energy addition will be discussed in the next section, Section 3.3.

### 3.3 Effects of energy addition ratio

Normalized thrust variation to energy addition ratio at the combustor is shown in Figure 6. As energy addition ratio increase, thrust performance increase constantly. Two conditions of $EA=0.2$, $s_{MHD}=-0.2$ and $EA=0.2$, $s_{MHD}=-0.2$ will be compared in this section for representative conditions. Pressure and velocity distributions are shown in Figure 7 and Figure 8 respectively.

![Fig.6. Normalized thrust to energy addition ratio at the combustor](image)

Higher pressure at the exit of the combustor was obtained with higher energy addition ratio. Because the flow is expanded from higher pressure, high exit pressure was realized. High exit pressure is essential for avoiding overexpansion which would cause thrust reduction. Using the equations (11) and (12), quantity of lorentz force could be described as below.

$$F = \frac{Q\eta_{acc}}{u} \quad (15)$$

Here, $Q$ is the amount of added energy and $u$ is the flow velocity. When equal amount of energy is input to the MHD accelerator, more effect of Lorentz force could be obtained with less flow velocity. Thus, more acceleration could be obtained at lower velocity. In the case of $EA=0.4$, flow is decelerated in the combustor by the effect of heating, but on the other hand, more effect of acceleration was obtained at the MHD accelerator. This resulted in higher velocity at the exit and in consequent higher thrust performance. This could be also confirmed from the Lorentz force. Lorentz force in the MHD accelerator could be compared through interaction parameter, since it is described as in Eqn.(1). The interaction parameter in the condition $EA=0.0$ was 0.13 while it was 0.15 when $EA=0.4$. 

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In summary, higher energy addition ratio caused both pressure and velocity increase. Therefore, increase in thrust performance was presented.

### 3.4 Effects of inlet Mach number

As it is discussed in Section 3.2, stagnation temperature in the combustor could be suppressed by introducing the MHD energy bypass system. In other words, more energy could be injected at the combustor with equivalent stagnation temperature. From the results obtained in Section 3.3, more thrust could be expected by increasing energy injection. In this section, the maximum engine exit stagnation temperature is set to 3000K and energy is injected at the combustor as much as possible. Normalized thrust to various inlet Mach number conditions were calculated with and without MHD energy bypass system.

The result indicated the existence of maximal thrust performance to inlet Mach number. Higher maximal thrust was obtained with interaction parameter of 0.00 at the MHD generator. On the other hand, higher thrust performance was obtained in the case of $s_{MHD}=-0.25$ at high Mach number conditions of $M\text{in}>6$. First, the reason of low performance at low Mach number will be discussed.

Figure 10 is the Mach number distribution at inlet Mach number 3 representing low Mach number conditions. In both conditions, any more energy input at the combustor would cause thermal choking in the combustor. Especially at the inlet of the combustor, lower Mach number was obtained due to MHD interaction at the generator in the case of $s_{MHD}=-0.25$, and less energy addition was possible to avoid thermal choking. Stagnation temperature at the exit of the combustor without and with MHD system was 1050K and 700K respectively. Therefore, the operating condition at low Mach number is not limited by the combustor exit stagnation temperature, but thermal choking. Minimum Mach number possible for MHD energy bypass is the condition which would cause choking in the MHD generator.
Next, thrust difference and engine characteristics at high Mach number regime is discussed. The stagnation temperature distributions at inlet Mach number of 7 is brought out in Figure 11 for comparison.

Because the inlet Mach number is high, thermal choking in the combustor is always avoided, but the combustor exit temperature is at the limit. Energy extraction at the MHD generator had effectively decreased the stagnation temperature prior to the combustor. Accordingly, greater thermal injection was made possible to meet the same engine limit temperature. The un-filled plots in the Figure 9 is the conditions which combustor exit stagnation temperature is at the limit of 3000K. It could be known that MHD energy bypass system has an advantage at these high Mach number conditions.

3.5 Comparison to analytical solutions

In this section, results obtained from present research and the results obtained from analytical calculation is compared. Method for analytical calculation was referred from the research by Litchford et al. Next assumptions were made in order to carry out analytical calculation.

- Isobaric energy addition at the combustor.
- Optimum expansion at the engine exit.

Also, the operating conditions at the MHD generator is described using enthalpy extraction ratio (EE). Enthalpy extraction ratio is defined as follows.

\[ EE = \frac{\text{Extracted enthalpy}}{\text{Thermal input}} \]  \hspace{1cm} (16)

Results obtained by both methods are shown in Figure 12.

Figure 12(a) is same figure as Figure 9, but the axis are changed to match the span in Figure 12(b). Tendency in both figures are similar, although the absolute value differs. The difference in the absolute value is thought to be the effect of channel dimensions, which is not yet optimized to match the analytical assumptions listed before. Some improvements to the channel dimensions will be discussed in Section 4. The major difference in the tendency is the Mach number which give the maximal thrust performance. Compared to present results, maximal thrust was obtained at lower Mach number in analytical calculation. The reason is thought as follows. In the analytical calculation, the effect of shock wave is not included, since supersonic and subsonic flow is calculated using the same equations. Thus, higher thrust was obtained at low Mach number where the flow was...
choked and energy input was limited in the quasi-one dimensional simulation.

4. Effects of channel configuration

The channel dimension considered in previous section had a strong tendency to perform overexpansion. Thus, it is difficult to obtain effective thrust to the traveling direction. Here, effects of engine channel dimension on thrust performance is discussed.

Three channels are compared in this section.

Channel named (1) is the channel used in previous section. Simple constant area channel is named (2), and the channel designed to perform isobaric combustion is named (3). Isobaric combustion is brought out to match the assumption in the analytical calculation. Also for channel (3), smaller exit cross sectional area was given, since the channel (1) tend to perform overexpansion.

Same calculating conditions are applied to all channel profiles as indicated in Table 3.

Table 3. Calculating conditions

<table>
<thead>
<tr>
<th>Calculating conditions</th>
<th>Figure 14. Flow distributions to various channel dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Mach number</td>
<td>5.0</td>
</tr>
<tr>
<td>Interaction parameter at MHD generator</td>
<td>-0.1</td>
</tr>
<tr>
<td>Energy addition ratio at combustor</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Pressure distributions are shown in Figure 14(a). In the case of original channel (1), high pressure is obtained in the combustor and steep pressure decrease in the MHD accelerator and the expansion nozzle could be observed due to rapid expansion. On the other hand, high pressure from the combustor to the exit is maintained in the case of channel (2), since the channel does not diverge. Constant pressure in the combustor was obtained using the channel (3) as designed. Because the exit cross sectional area is smaller than the channel (1), overexpansion is improved and relatively higher pressure was realized at the exit.

Referring to Mach number distribution in Figure 14(b), channel (1) and (2) has lower Mach number at the combustor since the cross sectional area does not increase rapidly in the combustor. Isobaric combustion channel (3) has less Mach number drop because no pressure gradient exists and no velocity drop occurs. This result implies the fact that isobaric combustion channel is effective for avoiding flow choking in the combustor. In other words, channel (3) has the potential for larger
energy input in consequent thrust increase before choking.

**Table 4. Exit properties and thrust**

<table>
<thead>
<tr>
<th>Channel type</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exhaust velocity [m/s]</td>
<td>1656</td>
<td>1326</td>
<td>1566</td>
</tr>
<tr>
<td>Exit pressure [kPa]</td>
<td>0.34</td>
<td>5.57</td>
<td>0.72</td>
</tr>
<tr>
<td>Thrust [N]</td>
<td>-23.3</td>
<td>-16.1</td>
<td>15.3</td>
</tr>
</tbody>
</table>

Velocity distributions are shown in Figure 14(c). Rapid expansion cause high exhaust velocity as in channel (1) and non expansion channel (3) has very low exit velocity. Relatively high velocity is obtained in channel (3) at the MHD accelerator since the velocity in the combustor does not drop. Exhaust velocity is lower than channel (1) due to smaller cross sectional area at the exit.

Exit properties are listed in Table 4. Channel (1) has the highest exit velocity and the channel (2) has especially high exit pressure. But in total, the isobaric combustion channel (3) had the highest thrust even though both velocity and pressure was moderate. It could be said from this result that a channel is needed to be designed to realize optimum expansion to avoid pressure decrease and obtain maximum momentum increase.

5. Conclusions

The characteristics and performance of MHD energy bypass scramjet engine was determined by employing quasi-one dimensional numerical simulation. The following conclusions were obtained.

1. Higher thrust performance was suggested at high Mach number conditions by introducing the MHD energy bypass system. Larger heat injection was obtained due to the effect of MHD generator suppressing the stagnation temperature prior to the combustor.
2. Thrust decrease was suggested at low Mach number conditions. Mach number reduction in the MHD generator caused to decrease heat injection at the combustor because of the choking limitations.
3. Good agreement with analytical results were obtained qualitatively. The difference in the maximal thrust Mach number is thought to be the effect of choking limits in the present simulation.
4. Strong dependency on engine channel dimension was pointed out. Improvement in thrust performance was implied by avoiding overexpansion.

References